MATH 623 Final Exam
Fall 2014

Please read the following instructions. For the following exam you are free to use a one-page formula sheet, but no other resources. Please turn in **exactly eight** problems. You must do problems 1,2,3,4,5 and three more chosen from the remainder. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 120 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 40 and 80. This will then be multiplied by 1.25 for your exam score.

Turn in problems 1, 2, 3, 4, 5:

For problems 1,2, let $A = \begin{pmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$.

1. Find the Jordan and Weyr canonical forms for A.
2. Find the characteristic and minimal polynomials for A. Find the companion matrix to the minimal polynomial.
3. Fix three real numbers $r < s < t$. For $f \in C[r,t]$, define $\|f\|_s = |f(s)|$. Prove that $\|\cdot\|_s$ is a seminorm on $C[r,t]$, but not a norm.
4. Let $x \in \mathbb{C}^n$. Prove that $\|x\|_\infty = \lim_{p\to\infty} \|x\|_p$, where $\|\cdot\|_p$ denotes the usual p-norm.
5. Let $|||\cdot|||$ denote a matrix norm on M_n, and let $c \geq 1$. Prove that $c|||\cdot|||$ is also a matrix norm.

Turn in exactly three more problems of your choice:

6. For $A = \left(\begin{smallmatrix} 1 & 1 \\ 1 & 2 \end{smallmatrix}\right)$, find orthogonal matrix Q and upper triangular matrix R such that $A = QR$.
7. For $B = \left(\begin{smallmatrix} 1 & 5 \\ 0 & 3 \end{smallmatrix}\right)$, find bases for each left and right eigenspace.
8. Let C be skew Hermitian. Prove that the eigenvalues of C^2 are all real and nonpositive.
9. Show that the vector norms $\|\cdot\|_1$ and $\|\cdot\|_\infty$ on $\mathbb{C}^n$ both fail to satisfy the parallelogram identity:
$$\text{for all } x, y \in V, \quad \|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$$
10. Let V be a normed vector space, and $S \subseteq V$. Prove that S is closed if and only if S contains all of its limit points.