MATH601 Spring 2008 Handout 4: Subtraction
Unit 1: Natural Numbers

Unless otherwise specified, everything on this handout is using exactly three hands. For example, AB really means ZAB (with the initial Z suppressed).

The complement of a number is found by swapping each hand: Z↔E, A↔D, B↔C. For example, the complement of AB is EDC.

The negative of a number is found by taking the complement and adding one. For example, the negative of A is EED+A=EEE. The negative of AB is EDC+A=EDD. Numbers beginning with Z,A,B (apart from zero) are called positive; those beginning with C,D,E are called negative. For example, D(=ZZD) is positive, EAA is negative.

To subtract (calculate the difference) $x - y$, for positive numbers x, y, we first compute negative y (i.e. $-y$), then add $x + (-y)$. We do NOT carry past the left side (anything carried there simply disappears); if we are working with three hands, the result will always be in three hands. For example, four minus one is D-A=D+EEE=ZZC=C.

If we were working in binary rather than senary, this method of representing negative numbers and subtraction is called two's complement. This is the way virtually all modern computers do things, because each operation (adding one, taking complements) is very fast. For more details, see:
`mathforum.org/library/drmath/view/54344.html`
`en.wikipedia.org/wiki/Two%27s_complement`

Exercises to complete for next class:

A. Calculate BB-AA, BB-AC, AC-BB, BAD-BEE, EAA-ABC, EAA+EAA, CAB-ABC. Express the completed problems as statements with spelled-out numbers.
(you will find these methods do not always give the correct answer!)

B. Find all numbers x such that $x = -x$, where $-x$ is calculated as above.

C. Prove that, for every number x, $x + (-x) = Z$, where $-x$ is calculated as above.

D. Prove that, for every number x, $-(-x) = x$, where we calculate negative twice as above.

E. With three hands, are there more positive numbers or negative numbers? For all $n \geq$A, find formulas for how many positive and negative numbers there are, with n hands.

F. With three hands, when will subtraction give us the WRONG answer? When will addition give us the WRONG answer? What about $n \geq$A hands?