MATH601 Spring 2008 Handout 3: Divisibility Tests
Unit 1: Natural Numbers

We recall divisibility tests in decimal arithmetic, and try to find analogous tests in senary.

The last digit tests:
The last digit is a multiple of {two, five, ten} iff[1] the number is a multiple of {two, five, ten}.

The last two digits tests:
The last two digits (treated as a two-digit number) is a multiple of {four,twenty-five,twenty,fifty,one hundred} iff the number is a multiple of {four,twenty-five,twenty,fifty,one hundred}.

The last three digits tests:
The last three digits are a multiple of {eight,one hundred twenty-five,forty,two hundred,two hundred fifty,five hundred,one thousand} iff the number is a multiple of {eight,one hundred twenty-five,forty,two hundred,two hundred fifty,five hundred,one thousand}.

What's the pattern? What are the last n-digits tests?

The sum-of-digits tests:
Add the digits to get a new number. If desired, repeat this process until a single digit remains. The resulting number is a multiple of {three,nine} iff the original number is a multiple of {three,nine}.

The alternating test:
Alternately add and subtract the digits, from the right, to get a new number. If desired, repeat this process until a single digit remains. The resulting number is a multiple of eleven iff the original number is a multiple of eleven. (ex: $468321 \to 1-2+3-8+6-4=-4$, not a multiple; $464321 \to 1-2+3-4+6-4=0$, a multiple of 11)

For more details and tests see:
`egge.net/~savory/maths1.htm`
`mathforum.org/library/drmath/view/58510.html`

Exercises to complete for next class:

A. Which number(s) does the last-hand test work for, in senary? Prove your answer.
B. Which number(s) does the last n-hand test work for, in senary, for all positive integer n? Prove your answer.
C. Which number(s) does the sum-of-hands test work for, in senary? Prove your answer.
D. Which number(s) does the alternating hands test work for, in senary? Prove your answer.
E. Make a list of all the integers up to fifty that we can test for using the above tests or combinations of those tests[2], in decimal. Make a similar list for senary. What are the integers we can test for in decimal that we can't test for in senary? What are the integers we can test for in senary that we can't test for in decimal?

For the proofs in exercises C and D, you may want to use the following factoring theorem, with $x = AZ, y = A$. (to prove the analogous decimal versions, you would use $x = 10, y = 1$)

THM:
- For any positive integer a, $x^a - y^a$ is a multiple of $x - y$.
- For any even positive integer a, $x^a - y^a$ is a multiple of $x + y$.
- For any odd positive integer a, $x^a + y^a$ is a multiple of $x + y$.

[1] iff means 'if and only if'; i.e. the implication goes both ways

[2] For example, in decimal we can test divisibility by six by testing for divisibility by two and divisibility by 3.