MATH601 Spring 2008
Final Exam

Please read all directions carefully. For this exam, you may use a calculator and a one-page note sheet. Please write all solutions clearly and legibly, on separate paper, indicating what work applies to which problem. Cross out incorrect statements, this may improve your grade. Thoroughly justify your solutions. Your grade on each problem will be 7-14 points, and you earn 2 points for signing your name. Choose seven of these eight problems. You may do all eight for extra credit. You have two hours; good luck!

Note: If you forgot any of the terms below, you may "buy" definitions for 2 points each.

1. Express the decimal number 12345 in senary (with the letter notation we used).
2. Prove or disprove that $\mathbb{R}$ is Archimedean.
3. Use the hyperreal definition of derivative to calculate $f'(x)$, for $f(x) = \sin(2x)$.
4. Find sets S, T such that $S = T \cup \{x\}$ for some single element $x \notin T$, and yet $|S| = |T|$.
5. State the usual definition of the axiom of choice, and give two other equivalent statements.
6. Prove that $x^y \times x^z = x^{y+z}$ for all ordinals x, y, z.
7. Prove or disprove that $\mathbb{S}$, the set of surreal numbers, is dense.
8. Label the three unlabeled edges of this position of Hackenbush so that the resulting bush has value $1/2$: