MATH601 Spring 2008
Exam 1 Solutions

A. Write nineteen and one hundred in senary, then multiply the results using the Russian Peasant method.

Nineteen is three times six, plus one: CA. One hundred is two times thirty six, leaving twenty eight. Twenty eight is four times six, plus six. Hence one hundred is BDD.

It's easier to put the nineteen on the left (five rows instead of seven):

CA	BDD	$\leftarrow$
AC	ECB	$\leftarrow$
D	AEZD	
B	CDAB	
A	AABBD	$\leftarrow$

BDD+ECB+AABBD=ABDDD (=CA×BDD)

B. Given a five-hand number n, let m be the alternating sum of the hands of n (adding the leftmost, middle, and rightmost hands, and subtracting the two others). Prove that seven divides n if and only if seven divides m.

Write $n = (AZ)^D n_D + (AZ)^C n_C + (AZ)^B n_B + (AZ)^A n_A + (AZ)^Z n_Z$, where n_i are the hands of n. We have $m = n_D - n_C + n_B - n_A + n_Z$, and $n - m = ((AZ)^D - A)n_D + ((AZ)^C + A)n_C + ((AZ)^B - A)n_B + ((AZ)^A + A)n_A$.

We now show that seven (AA) divides each of $(AZ)^D - A, (AZ)^C + A, (AZ)^B - A, (AZ)^A + A$. This can be done with the theorem on Handout 3: AA divides $(AZ)^x - A$ for even x, and AA divides $(AZ)^x + A$ for odd x. Or, this can be done by noting that these numbers are EEEE=AA×EZE, AZZA=AA×EA, EE=AA×E, AA=AA×A. Hence, seven divides $n - m$. If seven divides n, then seven divides $n - (n - m) = m$ as well. Similarly, if seven divides m, then seven divides $m + (n - m) = n$ as well.

C. Exam grades: BDC (ninety-nine), BDC (ninety-nine), BDB (ninety-eight), BDA (ninety-seven), BDZ (ninety-six), BCE (ninety-five), BBA (eighty-five), BAB (eighty), BAZ (seventy-eight), AED (seventy).