MATH 579 Exam 4; 3/1/12

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

Choose three problems only from these five.

1. (5-8 points) How many subsets of $[30]$ are larger than their complements?
2. (5-10 points) Prove that $\sum_k (-4)^k \binom{100}{k} = 3^{100}$.
3. (5-10 points) How many northeastern lattice paths are there from $(0,0)$ to $(15,10)$ that do not pass through $(7,8)$?
4. (5-10 points) Suppose k, n are integers satisfying $0 < k < n$. Prove that
$$\binom{n}{k-1}\binom{n}{k+1} \leq \binom{n}{k}\binom{n}{k}.$$
5. (5-12 points) For $m, n \in \mathbb{N}_0$, prove that
$$\sum_{0 \leq k \leq n} (k)_m = \frac{1}{m+1}(n+1)_{m+1}$$

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