Choose three problems only from these five.

1. (5-8 points) Using mathematical induction, prove that for all positive integers \( n \), we have \( 1 + 3 + 5 + \cdots + (2n - 1) = n^2 \).

2. (5-10 points) Let \( a_0 = a_1 = 1 \), and \( a_{n+2} = a_{n+1} + 5a_n \) for \( n \geq 0 \). Prove that \( a_n \leq 3^n \) for all \( n \geq 0 \).

3. (5-10 points) Given \( n \in \mathbb{N} \), the alternating sum of \( n \) is given as the units digit, minus the tens digit, plus the hundreds digit, minus the thousands digit, etc. For example, the alternating sum of 7,904,567 is \( 7 - 6 + 5 - 4 + 0 - 9 + 7 = 0 \). Prove that \( n \) is a multiple of 11 if and only if its alternating sum is a multiple of 11.

4. (5-10 points) Prove that for every triangulated simple polygon, it is possible to color each of its vertices red, blue, or green such that every triangle has its three vertices of different colors.

5. (5-12 points) The Fibonacci numbers are defined as \( F_1 = F_2 = 1 \), \( F_{n+2} = F_{n+1} + F_n \) for \( n \geq 1 \). Let \( \phi = \frac{1 + \sqrt{5}}{2} \). Prove that \( \frac{\phi^n}{\sqrt{5}} < F_n < \phi^n \), for all \( n \in \mathbb{N} \).