Part I: Recall the difference operator $\Delta$, where $\Delta f(x) = f(x + 1) - f(x)$. Define the shift operator $E$, as $Ef(x) = f(x + 1)$.

1. Consider functions $u(x), v(x)$. Prove the (difference calculus) product rule:
$$\Delta(uv) = u\Delta v + (Ev)\Delta u.$$ 

2. Use the product rule to prove the summation by parts formula:
$$\sum u\Delta v\delta x = uv - \sum (Ev)\Delta u\delta x.$$ 

3. Use summation by parts to find a closed form for $\sum_{k=0}^{n} k^2$.

NOTE: It is not necessary to do these parts in order, you may assume 1 to do 2; you may assume 2 to do 3.