

MATH 579 Exam 2 Part II; 2/16/10

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please write your answers on separate paper, indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this sheet for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. Please attach part I to your solutions. You have 40 minutes. If you wish, you may hand in solutions to all six problems (part I and II) on the next class day, February 18. For more details, see the syllabus.

PART II: Choose three problems only from these five.

1. (5-8 points) Consider the sequence $a_0 = 1, a_{n+1} = 3a_n + 2$ (for $n \geq 0$). Prove that $a_n = 2 \cdot 3^n - 1$.
2. (5-10 points) Consider the sequence $b_0 = 3.5, b_{n+1} = \sqrt{b_n + 7}$. Prove that $b_n \in (3, 4)$ for all $n \in \mathbb{N}_0$. (i.e. $3 < b_n < 4$)
3. (5-10 points) For all $n \in \mathbb{N}$, prove that $5 \mid 2^{2n-1} + 3^{2n-1}$.
4. (5-10 points) For all $n \in \mathbb{N}$, prove that $3 \mid n^3 + 11n$.
5. (5-12 points) Prove the Bernoulli inequality: if $1 + a > 0$, then $(1 + a)^n \geq 1 + na$ for all $n \in \mathbb{N}$.