Part I: In the coin game, we place $n$ coins in a row, each either heads or tails. We can remove any of the heads, leaving a gap, but when we do we flip any immediately adjacent coins. This means at most two coins get flipped, but often fewer because gaps destroy adjacency. We keep doing this until we run out of heads; we win if all the coins are gone. For example we could play \( THHH \rightarrow H \cdot TH \rightarrow H \cdot H \cdot \rightarrow \cdots \rightarrow \cdots \) (we win), or we could play as \( THHH \rightarrow TT \cdot T \) (we lose). Prove that it is possible to win if and only if the number of initial heads is odd.