MATH 579 Exam 1 Part II; 1/28/10

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please write your answers on separate paper, indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this sheet for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. Please attach part I to your solutions. You have 40 minutes. If you wish, you may hand in solutions to all six problems (part I and II) on the next class day, February 9. For more details, see the syllabus.

PART II: Choose three problems only from these five.

1. (5-8 points) An airport has 1500 takeoffs per day. Prove that there are two planes taking off within a minute of each other.
2. (5-10 points) Your nemesis chooses 1001 distinct integers from $[1, 2000]$. Prove that some pair of these must have no nontrivial (greater than 1) common factor.
3. (5-10 points) Your nemesis chooses 55 distinct integers in $[1, 100]$. Prove that some pair of these must differ by 12.
4. (5-10 points) Prove that the sequence $2010, 20102010, 201020102010, \ldots$ contains an element that is divisible by 2011.
5. (5-12 points) Your nemesis colors each point of 3-dimensional space one of red, blue, or green. Prove that there is some rectangular box (all six faces are rectangles, like a shoebox), such that all eight corners of this box are the same color.