

MATH 579 Exam 6: 4/9/9
Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting by the next class period (Apr. 14), revised solutions to all six problems – for more details, please see the syllabus. This exam is out of 40 points maximum.

PART I: Choose three problems only from the first five.

1. (5-8 points) Consider the permutation on $[4]$ given by $f(1) = 4, f(2) = 1, f(3) = 3, f(4) = 2$. Write this in two-line notation, one-line notation, canonical cycle form, and as a directed graph.
2. (5-10 points) For all $n \in \mathbb{N}$, prove that the number of partitions of n into no more than k parts is equal to the number of partitions of $n + 2k$ into exactly k parts, each of size at least 2.
3. (5-10 points) Consider the permutations on $[8]$ given by $\pi : (51234)(876)$, $\sigma : (2)(31)(745)(86)$. Calculate $\pi \circ \sigma \circ \pi^{-1}$.
4. (5-10 points) Find all self-conjugate partitions of 20.
5. (5-12 points) For all $n \in \mathbb{N}$, prove that $p(n)^2 < p(n^2 + 2n)$.

PART II: Choose either problem 6 or problem 7.

6. (5-10 points) Let X be the set of 19 students enrolled in this course, and let $Y = \{A, B, C, D, F\}$. I want to count how many ways to assign grades (from Y) to you all. Each of the following is WRONG; none of them count the desired quantity. For each, explain what it actually *IS* counting, including at least one example, and how this is different from what I'm trying to count. Finally, give the right answer. You need not calculate or simplify anything.
 - (a) $\binom{19}{5}$
 - (b) $\binom{5}{19}$
 - (c) $S(19, 5)$
 - (d) $5!S(19, 5)$
 - (e) $(19)_5$
 - (f) $p_5(19)$
7. Do both problems that you skipped from Part I. Your score will be the lower of the two. Be sure to indicate which two problems you are counting as problem 7.