

## Math 579 Final Exam: 5/10/7

Please read the exam instructions.

Books and notes are forbidden for this exam. Please write your answers on **separate paper** and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this page for your records. Show all necessary work in your solution; if you are unsure, show it. Each problem is worth 10-20 points. You have 60 minutes.

1. Fix positive integer  $m$ . Let  $A$  be an alphabet consisting of  $m$  different symbols. We add the special symbol  $X$ ; set  $A^+ = A \cup \{X\}$  and  $|A^+| = m + 1$ . We consider words with letters drawn from  $A^+$ . We call a word *legitimate* if it contains an even number (possibly 0) of  $X$  symbols. Let  $b_n$  represent the number of legitimate  $n$ -letter words with letters drawn from  $A^+$ . We have  $b_1 = m, b_2 = m^2 + 1$ . Find a linear nonhomogeneous constant-coefficient recurrence relation on  $b_n$ , and use it to find a closed form for  $b_n$ .
2. The ordinary way to label an eight-sided die is 1, 2, 3, 4, 5, 6, 7, 8. If we roll two ordinary eight-sided dice, and add the results, the totals are represented in the following table, together with their probabilities. Determine all other methods (if any) of labelling two eight-sided dice with positive integers, to have the same possible totals, with the same probabilities. HINT: The following polynomials are irreducible over the integers:  $x + 1, x^2 + 1, x^4 + 1, x^8 + 1$ .

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\frac{1}{64}$	$\frac{2}{64}$	$\frac{3}{64}$	$\frac{4}{64}$	$\frac{5}{64}$	$\frac{6}{64}$	$\frac{7}{64}$	$\frac{8}{64}$	$\frac{7}{64}$	$\frac{6}{64}$	$\frac{5}{64}$	$\frac{4}{64}$	$\frac{3}{64}$	$\frac{2}{64}$	$\frac{1}{64}$