Math 579 Exam 8 (part I): 4/24/7

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth a minimum of 5 points, and a maximum that is indicated. You have 40 minutes. *Choose three problems. Simplify all numerical answers.*

1. (8 points) Carefully define the following three terms: formal power series, (ordinary) generating function, partial fractions.

2. (10 points) Find a generating function that can be used to count how many a, b, c, d there are that solve $a + b + c + d = n$ and satisfy:
   (1) a, b, c, d are nonnegative integers,
   (2) a is 2,3, or 7, and
   (3) b, c are multiples of 3.
   NOTE: you do not need a closed form solution, merely a g.f.

3. (10 points) $A(x) = \frac{x^5+3x-2}{(1+x)^5}$ is the generating function for a sequence a_n. Find a closed form for a_n (for $n \geq 5$ is sufficient).

4. (10 points) $a_0 = 0, a_{k+1} = 2a_k + 2^k$. Using generating functions, find a closed form for a_k.

5. (12 points) $a_0 = 1, a_1 = 5, a_n = a_{n-1} + 2a_{n-2}$ $(n \geq 2)$. Using generating functions, find a closed form for a_n.