Math 579 Exam 4 (part I): 3/1/7
Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth a minimum of 5 points, and a maximum that is indicated. You have 40 minutes. Choose three problems.

1. (8 points) How many subsets of \([n]\) are larger than their complements?

2. (10 points) Evaluate the sum \(\sum_{k=0}^{n-1} \frac{1}{k+1} \binom{n}{k}\).

3. (10 points) Let \(n \in \mathbb{N}\). Prove that \(\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2\).

4. (10 points) We may write \(x^4 = (x)_4 + 6(x)_3 + a(x)_2 + (x)_1\), for some integer constant \(a\). First, find \(a\). Then, use the difference calculus to evaluate in closed form \(\sum_{k=0}^{n} k^4\).

5. (12 points) Let \(p\) be prime. Prove that \(p\) divides \((p-1)_k + (-1)^{k+1}\), for all \(k\) satisfying \(0 \leq k \leq p - 1\).

HINT: Start by proving that \(p\) divides \(\binom{p}{k}\) for all \(k\) with \(1 \leq k \leq p - 1\).