A graph $G = (V, E)$ consists of a set of vertices $V$ and a set of edges $E$. Each edge is a set consisting of a pair of vertices. Note: for us, an edge must contain two distinct vertices, and all edges must be different. This is often called a “simple graph” in the literature.

**incident** If edge $e$ contains vertex $v$, then we say each is incident with the other.

**adjacent** If edge $e$ contains vertices $u, v$, then we say that vertices $u, v$ are adjacent.

**degree** The degree of a vertex is the number of edges that is incident with it.

**walk** A walk is a list $v_0, e_1, v_1, e_2, \ldots, e_k, v_k$ where for $1 \leq i \leq k$, $e_i = \{v_{i-1}, v_i\}$. Its length is $k$.

**closed** A walk is closed if $v_0 = v_k$.

**trail** A trail is a walk with no edge repeated.

**path** A path is a walk with no edge or vertex repeated.

**cycle** A cycle is a closed path.

**even graph** A graph is even if all of its vertices are of even degree.

**Eulerian** A closed trail is Eulerian if it contains every edge of the graph. A graph is Eulerian if it has an Eulerian trail.

**Hamiltonian** A path or cycle is Hamiltonian if it contains every vertex of the graph.

**subgraph** $G' = (V', E')$ is a subgraph of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$.

**connected** A graph is connected if there is a path between any pair of vertices.

**component** A component of a graph is a maximal connected subgraph. A component is nontrivial if it has at least one edge.

$K_n$ The complete graph $K_n$ consists of $n$ vertices and every possible edge between them.

**clique, coclique** A clique is a complete subgraph. A coclique is a set of vertices containing no edges between them.

**bipartite** Graph $G = (V, E)$ is bipartite if there is a partition $V = V_1 \cup V_2$ and every edge contains exactly vertex from $V_1$ and one from $V_2$.

$K_{m,n}$ The complete bipartite graph $K_{m,n}$ has partition $V = V_1 \cup V_2$ with $|V_1| = m$, $|V_2| = n$, and every possible edge between $V_1$ and $V_2$.

$C_n$ The cycle graph $C_n$ contains $n$ vertices, edges to form a cycle of length $n$, and nothing else.

**Petersen graph** The vertices are the two-element subsets of $\{a, b, c, d, e\}$. An edge contains $\{u, v\}$ with $\{x, y\}$ if these two sets are disjoint.

**decomposition** A decomposition of a graph is a partition of the edges (each part forms a subgraph).

**graph isomorphism** Given graphs $G = (V, E)$ and $G' = (V', E')$, an isomorphism from $G$ to $G'$ is a bijection $f : V \rightarrow V'$ satisfying the property $\{u, v\} \in E \iff \{f(u), f(v)\} \in E'$.

**tree** A tree is a connected graph containing no cycles.

**pendant** A vertex is pendant (also called a leaf) if it has degree 1.

**spanning tree** A spanning tree is a subgraph, on all the vertices, that is also a tree.

*Singular of “vertices” is vertex.*