1. Prove the following properties for arbitrary constant $C$ and functions $f(x), g(x)$.
   (a) $\Delta C = 0$;
   (b) $\Delta (C f(x)) = C \Delta f(x)$; and
   (c) $\Delta (f(x) + g(x)) = (\Delta f(x)) + (\Delta g(x))$.
2. Find all functions $f(x)$ satisfying $\Delta (\Delta f(x)) = 3$.
3. Compute $\sum_{i=1}^{n} i^5$, for arbitrary $n \in \mathbb{N}$.
4. Let $c \in \mathbb{R}$. Compute $\Delta c^x$. Use this to find an anti-difference of $c^x$, and hence the geometric sum $\sum_{a}^{b} c^x \delta x$ (for $c \neq 1$).
5. For $c \in \mathbb{R}$ and $x \in \mathbb{N}$, compute $\Delta c^x$. Use this to find an anti-difference of $\sum_{k=2}^{n} \frac{(-2)^k}{k}$, and hence the sum $\sum_{k=2}^{n} \frac{(-2)^k}{k}$.
6. For $k \in \mathbb{N}$, we define $x^{-k} = \frac{1}{(x+1)(x+2) \cdots (x+k)}$. Prove that $\Delta x^{-k} = -k x^{-k-1}$.
7. For $x \in \mathbb{N}$, we define $H_x = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{x}$. Henceforth we may consider $H_x$ to be a basic function, in “closed form”. Prove that $\Delta H_x = x^{-1}$.
8. Prove that $x^{m+n} = x^{m}(x - m)^{n}$ for all integers $m, n$. (there are cases)
9. Calculate $\sum_{0}^{n} x 3^x \delta x$. Your answer should be a function of $n$.
10. Calculate $\sum_{0}^{n} x 2^x \delta x$.
11. Calculate $\sum_{0}^{n} x H_x \delta x$. (hint: summation by parts and exercise 8)
12. Calculate $\sum_{1}^{n} \frac{2k + 1}{k(k+1)}$. 

Please solve these problems using the methods of difference calculus (as presented in class).