MATH 579: Combinatorics
Homework 2: Due Sep. 11

1. Calculate $S(5,3)$ in two ways: with the formula involving binomial coefficients, and with the recurrence relation (and boundary conditions).
2. Explicitly find all partitions of $\{a,b,c,d,e\}$ into three nonempty parts.
3. Explicitly find all lists of length four, drawn from $[3]$, using each of $1,2,3$ at least once.
4. Explicitly find all partitions of $\{a,b,c,d\}$ into any number of parts.
5. Explicitly find all lists of length three, drawn from $[n]$ for some $n \in \mathbb{N}$, using each of $1,2,\ldots,n$ at least once*.
6. Determine the number of factorizations of 2310 into integers greater than 1. For example, 2310 and $2 \cdot 1155$ are two of these.
7. Prove the boundary conditions $S(n,1) = S(n,n) = 1$, for all $n \in \mathbb{N}$.
8. Prove the recurrence relation $S(n+1,k) = kS(n,k) + S(n,k-1)$, for $n \geq k \geq 1$.
   Hint: look at the element $n+1$ separately.
9. Prove that there are $n!S(k,n)$ lists of size k, drawn from $[n]$, using each of $1,2,\ldots n$ at least once.
10. Prove that $x^n = \sum_{k=1}^{n} S(n,k)x^{\underline{k}}$, for all $n \in \mathbb{N}$.
    Hints: Induction on n, and $x = (x-k)+k$.

*In this course, $\mathbb{N} = \{1,2,3,\ldots\}$, while $\mathbb{N}_0 = \{0,1,2,3,\ldots\}$.