MATH 579: Combinatorics Exam 3

Please read the following instructions. For the following exam you may not use any papers, books, or computers. You may use a calculator. Please turn in exactly four problems. You must do problems 1-3, and one more chosen from 4-6. Number 7 is optional. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 50 minutes. Be sure to adequately justify all your solutions. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 20 and 40. This will then be multiplied by $\frac{5}{2}$ for your exam score.

Turn in problems 1, 2, 3:

1. Use a characteristic equation (not generating functions) to solve the following recurrence. 
   $a_0 = 0, a_1 = 9, a_n = -6a_{n-1} - 9a_{n-2}$ $(n \geq 2)$.

2. Use generating functions to solve the following recurrence. 
   $a_0 = 0, a_1 = 9, a_n = -6a_{n-1} - 9a_{n-2}$ $(n \geq 2)$.

3. Use inclusion/exclusion to find the chromatic polynomial for: 
   $ab$ 
   $cd$ 

Turn in exactly one more problem of your choice:

4. Solve the following recurrence however you like: $a_0 = 0, a_n = 3a_{n-1} + 2^n + 3^n$ $(n \geq 1)$.

5. Count the number of solutions to $a + b + c + d = n$ in nonnegative integers $a, b, c, d$, such that $a$ is a multiple of 4, $b$ is at most 1, and $d$ is either 0 or 2.

6. Count the number of solutions to $a + b + c + d = 30$ in nonnegative integers $a, b, c, d$, such that $a \leq 9, b \leq 9, c \leq 9, d \leq 14$.

You may also turn in the following (optional):

7. Describe your preferences for your next group assignment. (will be kept confidential)