MATH 579 Exam 9; 12/5/13

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

Choose three problems only from these five.

1. (5-8 points) Describe a process that yields a sequence whose generating function is given by $\frac{1}{1-\frac{x}{(1-x)^2}}\left(=\frac{(1-x)^2}{x^2-3x+1}\right)$. [Note: no need to seek a closed form.]
2. (5-10 points) Let $k \in \mathbb{N}_0$, and consider the sequence given by $a_n = \binom{n+k}{k}$. Prove that it has generating function $A(x) = \frac{1}{(1-x)^{k+1}}$.
3. (5-10 points) Consider the sequence $a_n = n^2 + 2n + 3 + (-1)^n$ for $n \geq 0$. Find and simplify the generating function for this sequence.
4. (5-10 points) Use generating functions to solve the recurrence specified by $a_0 = 2, a_n = 3a_{n-1} + 2$ $(n \geq 1)$.
5. (5-12 points) When faced with a stack of exams to grade, I give each exam in turn either a pass or a fail. At some point (perhaps before I even start), my booze runs out and I stop grading immediately. The last thing I do is choose one of the remaining exams (and there is always at least one exam left), on which I draw a smiley face. Let a_n represent the number of ways I can do this with n exams. For example, $a_0 = 0$ (can't draw a smiley), $a_1 = 1$, $a_2 = 4$. Find a closed form for a_n.