Choose three problems only from these five.

1. (5-8 points) 55 distinct integers are selected, all from $[1, 100]$. Prove that some pair differs by 12.

2. (5-10 points) 300 points are placed within a unit cube. Prove that you can choose some 12 of these, all within 0.6 of each other.

3. (5-10 points) Use the PHP to prove that there is some $n \in \mathbb{N}$ such that $44^n - 1$ is divisible by 13.

4. (5-10 points) 17 distinct integers are selected, all from $[1, 33]$. Prove that some pair among these has greatest common divisor 1.

5. (5-12 points) Prove that there is a positive integer $n$ such that the distance from $n\pi$ to the nearest integer is less than $10^{-100}$. 