

**Math 524 Exam 8: 11/13/8**

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

All the problems concern the vector space  $\mathbb{R}_2[t]$  and the bilinear real symmetric form  $\langle f|g \rangle = \int_0^1 f(t)g(t)dt$ .

1. Under the standard basis  $E = \{1, t, t^2\}$ , find the metric  $G_E$ .
2. Prove that the above form is a (real) inner product.

The last two problems refer to the vectors  $u(t) = t - 1, v(t) = t^2 - 1$ . Set  $V = \text{Span}(u, v) = \{at^2 + bt - (a + b)\}$ .

3. Find an orthogonal basis for  $V$ .
4. For basis  $B = \{u, v\}$ , calculate two bases for  $V^*$  by specifying their action on each element of  $V$ . (1) the dual basis  $\{\phi_1, \phi_2\}$ , (2) the bra basis  $\{\langle u|, \langle v|\}$ .