Math 524 Exam 8: 11/13/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

All the problems concern the vector space $\mathbb{R}_2[t]$ and the bilinear real symmetric form $\langle f | g \rangle = \int_0^1 f(t)g(t)dt$.

1. Under the standard basis $E = \{1, t, t^2\}$, find the metric $G_E$.

2. Prove that the above form is a (real) inner product.

The last two problems refer to the vectors $u(t) = t - 1, v(t) = t^2 - 1$.
Set $V = \text{Span}(u, v) = \{at^2 + bt - (a + b)\}$.

3. Find an orthogonal basis for $V$.

4. For basis $B = \{u, v\}$, calculate two bases for $V^*$ by specifying their action on each element of $V$. (1) the dual basis $\{\phi_1, \phi_2\}$, (2) the bra basis $\{\langle u \rangle, \langle v \rangle\}$.