Math 524 Exam 6: 10/23/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

The first three problems all concern $A = \begin{pmatrix} -1/3 & -1/6 \\ 1/3 & -5/6 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -1/2 & 0 \\ 0 & -2/3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

1. Solve the discrete-time system given by $x(n) = Ax(n-1)$, with initial condition $x(0) = \binom{0}{1}$.

2. Solve the first-order system given by $\frac{d}{dt}x = Ax$, with initial condition given by $x(0) = \binom{0}{1}$.

3. Solve the second-order system given by $\frac{d^2}{dt^2}x = Ax$, with initial conditions given by $x(0) = \binom{0}{1}$ and $\dot{x}(0) = \binom{1}{0}$.

The last problem concerns $A = \begin{pmatrix} -2 & 1/2 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1/2 & 1/4 \\ -1/2 & 1/4 \end{pmatrix}$.

4. Solve the first-order system given by $\frac{d}{dt}x = Ax$, with initial condition given by $x(0) = \binom{0}{1}$.