Math 524 Exam 6: 10/23/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

The first three problems all concern $A = \begin{pmatrix} -1/3 & -1/6 \\ 1/3 & -5/6 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -1/2 & 0 \\ 0 & -2/3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$.

1. Solve the discrete-time system given by $x(n) = Ax(n-1)$, with initial condition $x(0) = (0 \ 1)$.

2. Solve the first-order system given by $\frac{dx}{dt} = Ax$, with initial condition given by $x(0) = (0 \ 1)$.

3. Solve the second-order system given by $\frac{d^2x}{dt^2} = Ax$, with initial conditions given by $x(0) = (0 \ 1)$ and $\dot{x}(0) = (\frac{1}{2} \ 0)$.

The last problem concerns $A = \begin{pmatrix} -2 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$.

4. Solve the first-order system given by $\frac{dx}{dt} = Ax$, with initial condition given by $x(0) = (0 \ 1)$. 