Math 522 Final Exam: 5/14/13

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Notes, calculators, and the textbook are all permitted. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. You will earn between 7 and 14 points on each problem (and a 2 point bonus because you're so awesome). You have 120 minutes.

Choose 7 of the following 8 problems to complete. You may do all 8 for a bonus.

1. [3.1.6] Suppose $r, n \in \mathbb{Z}$ and $0 < r \leq n$. Prove that $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^r \binom{n}{r} = (-1)^r \binom{n-1}{r}$.
2. [5.2.6] What is the remainder when 473^{38} is divided by 5?
3. [6.2.8] Prove that $d(n) < 2\sqrt{n}$.
4. Use the Euclidean algorithm to find an integer solution to $105x + 33y = 3$.
5. Use continued fractions to find the two smallest positive integer solutions to $x^2 - 6y^2 = 1$.
6. Let $p \geq 5$ be prime, and let g be a primitive root modulo p. Prove that $g^{\frac{p+1}{2}} \equiv -g \pmod{p}$.
7. For p an odd prime, determine whether or not $\frac{p-1}{2}$ is a quadratic residue modulo p, based on properties of p. (Hint: $\frac{p-1}{2}2 = p - 1$)
8. Let $n > 2$ be an integer, let S be a reduced residue system modulo n, and let $r \in S$. Prove that there is some $s \in S$ such that $n|(rs - 1)$.