Math 522 Final Exam: 12/13/7

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Notes, calculators, and the textbook are all permitted. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. You will earn between 7 and 14 points on each problem (and a 2 point bonus). You have 120 minutes. Choose seven of the following eight problems. If you do all eight, you will get the highest seven scores.

You may earn extra credit by submitting a revised answer to one of the following problems, by the end of the day on Monday, Dec. 17. Your score on that problem will be the average of the original score, and the revised score (rounded down).

1. (2-1:7) Prove that if a is an odd integer, then 12 divides $a^2+(a+2)^2+(a+4)^2+1$.
2. (5-2:21) Prove that if p is an odd prime, then $2^{\frac{p-1}{2}} \equiv \pm 1 \pmod{p}$.
3. (6-2:14) Assume Goldbach's conjecture (every even number > 2 may be written as the sum of two primes). Prove that for every even number $2m$ (with $m \in \mathbb{N}$) there exist integers m_1, m_2 with $2m = \sigma(m_1)+\sigma(m_2)$.
4. (6-4:9) Let $A(d,e)$ be some unknown function defined on all integer d, e. For all $n \in \mathbb{N}$, prove that
$$\sum_{d|n} \sum_{e|\frac{n}{d}} A(d,e) = \sum_{e|n} \sum_{d|\frac{n}{e}} A(d,e)$$
5. (7-2:9) For integer a and odd prime p with $p \nmid a$, prove that $a^{p^m - p^{m-1}} \equiv 1 \pmod{p^m}$.
6. Factor $\binom{28}{12}$.
7. Find all solutions to $5x^{2^{1000}} \equiv 8 \pmod{9}$.
8. Find an integer that is simultaneously congruent to $5 \pmod{7}$, and congruent to $7 \pmod{1000}$.