# Math 522 Exam 3: 9/20/7
Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Notes, calculators, and the textbook are all permitted. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. You will earn between 25 and 50 points on each problem. You have 30 minutes.

You may earn extra credit by submitting a revised answer to one of the following problems, by the next class day (9/25/7). Your score on that problem will be the average of the original score, and the revised score (rounded down).

1. In the country of Vadimia, license plates all contain exactly five characters. The first three must be letters (A-Z), but the last two two may be either letters (A-Z) or numbers (0-9). No letter may appear twice; numbers may be repeated however. How many license plates are possible?

2. Let $E = \{2k : k \in \mathbb{Z}\}$, the set of even integers, under the multiplication operation. We call $m \in E$ *irreducible* if there are no $a, b \in E$ with $m = ab$. (Note: the book calls this "even prime").

   (a) For every odd integer $k$, prove that $2k$ is irreducible.

   (b) Find two different factorizations of $2^2 \cdot 3 \cdot 5 \cdot 7(= 420)$ into irreducibles.

   (c) For every odd integer $k > 1$, set $x = 2k, y = 2k^2, z = 2$. Prove that $x|yz$ but $x \nmid y$ and $x \nmid z$. (Recall that $a|b$ means that $b/a \in E$)

   BONUS: Improve on (b), if possible: find the smallest positive integer in $E$ that has at least two different factorizations into irreducibles.