

Math 522 Exam 1: 9/6/7

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Notes, calculators, and the textbook are all permitted. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. You will earn between 25 and 50 points on each problem. You have 30 minutes.

You may earn extra credit by submitting a revised answer to one of the following problems, by the next class day (9/11/7). Your score on that problem will be the average of the original score, and the revised score (rounded down).

1. Please write one hundred in eight ways: in base 8,9,10,11,12,13,14,15. If needed, use 'A' to represent the digit ten, 'B' for eleven, and so on. BONUS: write one hundred in factoradic.
2. A nonempty set of integers J that fulfills the following two conditions is called an integral ideal:
 - (a) If n, m are in J , then $n + m$ and $n - m$ are in J ; and
 - (b) If n is in J and r is any integer, then rn is in J .

Further, for any integer m let $J_m = \{mk : k \in \mathbb{Z}\}$, the set of all integer multiples of m . You may assume that J_m is an integral ideal.

Prove that every integral ideal J is, in fact, equal to J_m for some $m \in \mathbb{Z}$.