1. For each of the following five shapes in $\mathbb{R}^2$, the isometry group is isomorphic to a subgroup of the permutation group on the vertices. Draw the shape, label the vertices, and give a set of generators for the abovementioned subgroup.

A: scalene triangle  
B: isosceles (not equilateral) triangle  
C: equilateral triangle  
D: rectangle (not square)  
E: square

2. We call a permutation a “cycle” if its cycle notation contains a single cycle (and all other elements are fixed). Prove that disjoint cycles commute.

3. Prove that every permutation may be written uniquely as a product of disjoint cycles. We call the number of each cycle length in this unique expression the “cycle structure” of the permutation. For example, $(1\ 2)(3\ 4)$ has two cycles of length 2.

For $a, b \in S_n$, we define $a^b = bab^{-1}$.

4. For $a = (1\ 2\ 3)(4\ 5), b = (1\ 4)(3\ 5)$, calculate $a^b$ and $b^a$.

5. Suppose $a, b \in S_n$ and $a = (a_1\ a_2\ \cdots\ a_k)$, a cycle. Prove that $a^b = (b(a_1)\ b(a_2)\ \cdots\ b(a_k))$.

6. Suppose $a, b \in S_n$. Prove that $a$ and $a^b$ have the same cycle structure.

7. Suppose $a, c \in S_n$. Suppose further that $a, c$ have the same cycle structure. Prove that there is some $b \in S_n$ such that $c = a^b$.

8. Let $a, c \in S^n$. If there is some $b \in S_n$ such that $c = a^b$, we say that $a, c$ are conjugate and write $a \sim c$. Prove that this is an equivalence relation, i.e.

(1) $x \sim x$  
(2) if $x \sim y$ then $y \sim x$  
(3) if $x \sim y$ and $y \sim z$ then $x \sim z$