MATH 521B: Abstract Algebra Exam 1

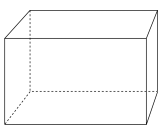
Please read the following instructions. For the following exam you are free to use a calculator and any papers you like, but no books or computers. Please turn in **exactly six** problems. You must do problems 1-4, and two more chosen from 5-8. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 75 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 30 and 60. This will then be multiplied by $\frac{5}{3}$ for your exam score.

Turn in problems 1,2,3,4:

1. Fix a group G. Let $a, b \in G$ with $ab = ba$, $|a|, |b|$ finite, and $\langle a \rangle \cap \langle b \rangle = \{id\}$. Prove that $|ab| = lcm(|a|, |b|)$.
2. Prove that every group G with exactly three elements must be abelian.
3. Let G be an abelian group. Let $f : G \to G$ be defined via $f : x \mapsto x^2$. Prove that f is a homomorphism.
4. Fix groups G, H, and a homomorphism $f : G \to H$. Let K be a subgroup of H. Prove that $S = \{g \in G : f(g) \in K\}$ is a subgroup of G.

Turn in exactly two more problems of your choice:

5. Fix a group G, a subgroup H, and an element $a \in G$. Define $aHa^{-1} = \{aha^{-1} : h \in H\}$. Prove that aHa^{-1} is a subgroup of G.
6. Fix a group G. Recall the group center $Z(G) = \{a \in G : \forall b \in G, ab = ba\}$, and the centralizer $C(x) = \{y \in G : xy = yx\}$, which is defined for each $x \in G$. Prove that $Z(G) = \bigcap_{g \in G} C(g)$.
7. Fix a group G. For subgroups A, B, we define their product $AB = \{ab : a \in A, b \in B\}$. Suppose H, K are both subgroups of G that satisfy $HK = KH$. Prove that HK is a subgroup of G.
8. Consider a solid square prism, i.e. shoebox, with two identical square ends and four identical rectangular (but not square) sides, as pictured below. Color each face black or white. How many different ways are there to do this, up to physically possible isometries of the solid figure?



You may also turn in the following (optional):

9. Describe your preferences for your next group assignment. (will be kept confidential)