1. Let \( R, S, T \) be rings, with \( S, T \) both subrings of \( R \). Suppose that \( S \) has the special property that for every \( s \in S \) and every \( r \in R \), we have both \( sr \in S \) and \( rs \in S \). Set \( S + T = \{ s + t : s \in S, t \in T \} \), a subset of \( R \). Prove that \( S + T \) is a subring of \( R \). [This is really a Chapter 3 question.]

2. Consider the polynomial ring \( \mathbb{Z}_9[x] \), and the nine elements \{3\( x \) + 0, 3\( x \) + 1, \ldots , 3\( x \) + 8\}. Determine which are units and which are zero divisors.

3. Consider the polynomial ring \( \mathbb{Z}_9[x] \), and the nine elements \{0\( x \) + 3, 1\( x \) + 3, \ldots , 8\( x \) + 3\}. Determine which are units and which are zero divisors.

4. Let \( R \) be a ring, and \( k \in \mathbb{N} \). Define \( x^k R[x] = \{ x^k f(x) : f(x) \in R[x] \} \). Prove that \( x^k R[x] \) is a subring of \( R[x] \).

5. Let \( F \) be a field. Determine explicitly which elements of \( F[x] \) are in the subring \( x^3 F[x] + x^5 F[x] \). (refer to exercises 1,4)

6. Working in \( \mathbb{Q}[x] \), find \( \gcd(a(x), b(x)) \), for \( a(x) = x^3 + x^2 + x + 1, \ b(x) = x^4 - 2x^2 - 3x - 2 \).

7. Working in \( \mathbb{Z}_2[x] \), find \( \gcd(a(x), b(x)) \), for \( a(x) = x^3 + x^2 + x + 1, \ b(x) = x^4 - 2x^2 - 3x - 2 \).

8. Working in \( \mathbb{Z}_5[x] \), find \( \gcd(a(x), b(x)) \), for \( a(x) = x^3 + x^2 + x + 1, \ b(x) = x^4 - 2x^2 - 3x - 2 \).

9. Working in \( \mathbb{Q}[x] \), let \( a(x) = x^2 - 5x + 6, \ b(x) = x^3 - x^2 - 2x \). Find \( u(x), v(x) \) such that \( \gcd(a(x), b(x)) = a(x)u(x) + b(x)v(x) \).

10. Working in \( \mathbb{Z}_3[x] \), let \( a(x) = x^2 - 5x + 6, \ b(x) = x^3 - x^2 - 2x \). Find \( u(x), v(x) \) such that \( \gcd(a(x), b(x)) = a(x)u(x) + b(x)v(x) \).