1. Let \( a, b \in \mathbb{N} \), and set \( d = \gcd(a, b) \). Prove that \( \gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1 \).

2. Let \( a, b, c \in \mathbb{Z} \). Consider the following equation (in variables \( x, y \)):
   \[
   ax + by = c
   \]
   Prove that this equation has integer solutions, if and only if \( \gcd(a, b)|c \).

3. Use the Generalized Euclidean Algorithm to find \( \gcd(196, 308) \) and also to find integers \( x, y \) satisfying \( 196x + 308y = \gcd(196, 308) \).

4. Let \( a, b \in \mathbb{N} \). Prove that the Euclidean Algorithm will find \( \gcd(a, b) \) in at most \( \min(a, b) \) steps.

5. Find all primes between 1025 and 1075.

6. Let \( a, b, n \in \mathbb{N} \). Prove that \( a|b \) if and only if \( a^n|b^n \).

7. Let \( n, k \in \mathbb{N} \) and let \( p \in \mathbb{N} \) be prime. Prove that if \( p|n^k \) then \( p^k|n^k \).

8. Let \( n \in \mathbb{N} \). Prove that \( n \) has an odd number of positive factors, if and only if, \( n \) is a perfect square.

9. Use the Miller-Rabin test on \( n = 69 \). Either find a witness to its compositeness, or else three potential liars.

10. Use the Miller-Rabin test on \( n = 66683 \). Either find a witness to its compositeness, or else three potential liars.