MATH 521A: Abstract Algebra
Homework 2: Due Sep. 13

1. Let $a, b \in \mathbb{N}$, and set $d = \gcd(a, b)$. Prove that $\gcd(\frac{a}{d}, \frac{b}{d}) = 1$.
2. Let $a, b, c \in \mathbb{Z}$. Consider the following equation (in variables x, y):

   $$ax + by = c$$

   Prove that this equation has integer solutions, if and only if $\gcd(a, b)|c$.
3. Use the Generalized Euclidean Algorithm to find $\gcd(196, 308)$ and also to find integers x, y satisfying $196x + 308y = \gcd(196, 308)$.
4. Let $a, b \in \mathbb{N}$. Prove that the Euclidean Algorithm will find $\gcd(a, b)$ in at most $\min(a, b)$ steps.
5. Find all primes between 1025 and 1075.
6. Let $a, b, n \in \mathbb{N}$. Prove that $a|b$ if and only if $a^n|b^n$.
7. Let $n, k \in \mathbb{N}$ and let $p \in \mathbb{N}$ be prime. Prove that if $p|n^k$ then $p^k|n^k$.
8. Let $n \in \mathbb{N}$. Prove that n has an odd number of positive factors, if and only if, n is a perfect square.
9. Use the Miller-Rabin test on $n = 69$. Either find a witness to its compositeness, or else three potential liars.
10. Use the Miller-Rabin test on $n = 66683$. Either find a witness to its compositeness, or else three potential liars.