MATH 521A: Abstract Algebra
Homework 9: Due Nov. 30

1. Find the equivalence classes, and rules for addition and multiplication, in $\mathbb{Q}[x]/(x^2-2)$.
2. Find the equivalence classes, and rules for addition and multiplication, in $\mathbb{Q}[x]/(x^2)$.
3. Find the equivalence classes, and rules for addition and multiplication, in $\mathbb{Q}[x]/(x^2+1)$.
4. For exercises 1-3, find all the units and zero divisors.
5. For exercises 1-3, find the inverse of $[3x-2]$ (in each respective ring).
6. Find a zero divisor in $\mathbb{Z}_2[x]/(x^4+x^2+1)$.
7. If $f(x) \in F[x]$ has degree n, prove that there is an extension field E of F so that $f(x)$ splits. That is, $f(x) = c_0(x-c_1)(x-c_2)\cdots(x-c_n)$ for some (not necessarily distinct) $c_i \in E$. Prove that the degree of E over F is at most $n!$.
8. Let $f(x) = x^3+x+1$, and set $E = \mathbb{Z}_2[x]/(x^3+x+1)$. Prove that $f(x)$ splits in E. That is, find three distinct roots of $f(x)$ in E.
9. Find a field with eight elements, and give the addition and multiplication table.
10. Prove that:
    (a) $2\cos\frac{2\pi}{5} = e^{2\pi i/5} + e^{-2\pi i/5}$ satisfies $x^2+x-1=0$; and
    (b) $2\cos\frac{2\pi}{7} = e^{2\pi i/7} + e^{-2\pi i/7}$ satisfies $x^3+x^2-2x-1=0$.
11. Use Problem 10 to prove that the regular pentagon is constructible with straightedge and compass, while the regular septagon (seven edges) is not.