1. Let $R$ be a ring, with additive and multiplicative neutral elements $0_R, 1_R$. Prove that $0_R, 1_R$ are unique.

2. For prime $p$, set $\mathbb{Z}[\sqrt{p}] = \{a + b\sqrt{p} : a, b \in \mathbb{Z}\}$. Prove that $\mathbb{Z}[\sqrt{p}]$ is a subring of $\mathbb{R}$.

3. For prime $p$, set $\mathbb{Q}[\sqrt{p}] = \{a + b\sqrt{p} : a, b \in \mathbb{Q}\}$. Prove that $\mathbb{Q}[\sqrt{p}]$ is a subfield of $\mathbb{R}$.

4. For $k \in \mathbb{Z}$, define object $R_k$, which has ground set $\mathbb{Z}$, and operations $\oplus, \odot$ defined as:

\[ a \oplus b = a + b, \quad a \odot b = k \]

Determine for which $k$, if any, $R_k$ is a ring.

5. Prove or disprove: If $R, S$ are fields, then $R \times S$ is an integral domain.

6. Define $R$, an object with ground set $\mathbb{Z}$, and operations $\oplus, \odot$ defined as:

\[ a \oplus b = a + b - 1, \quad a \odot b = a + b - ab \]

Prove that $R$ is an integral domain.

7. Define $R$, an object with ground set $\mathbb{Z}$, and operations $\oplus, \odot$ defined as:

\[ a \oplus b = a + b - 1, \quad a \odot b = ab - a - b + 2 \]

Prove that $R$ is an integral domain.

8. Define $R$, an object with ground set $\mathbb{Z} \cup \{+\infty\}$, and operations $\oplus, \odot$ defined as:

\[ a \oplus b = \min(a, b), \quad a \odot b = a + b \]

Prove that $R$ satisfies every field axiom except one, and prove that $R$ fails to satisfy that one.