MATH 521A: Abstract Algebra

Homework 3: Due Sep. 14

1. Write the $\oplus$-addition and $\odot$-multiplication tables of $\mathbb{Z}_{10}$.

2. For $\mathbb{Z}_{10}$, find the neutral additive element[1], the neutral multiplicative element[2], and all zero divisors[3].

3. Find the units of $\mathbb{Z}_{10}$; for each unit specify its inverse.

4. The *additive order* of an element in $\mathbb{Z}_{10}$ is the number of times one must $\oplus$-add it to itself to get $[0]$. Determine the additive order of each element of $\mathbb{Z}_{10}$.

We define $\mathbb{Z}_2 \times \mathbb{Z}_5 = \{(a, b) : a \in \mathbb{Z}_2, b \in \mathbb{Z}_5\}$, the set of ordered pairs of elements, one each from $\mathbb{Z}_2$ and $\mathbb{Z}_5$. We define operations in the natural way, i.e. componentwise: $(a, b) \oplus (a', b') = (a \oplus_2 a', b \oplus_5 b')$ and $(a, b) \odot (a', b') = (a \odot_2 a', b \odot_5 b')$.

5. Write the $\oplus$-addition and $\odot$-multiplication tables of $\mathbb{Z}_2 \times \mathbb{Z}_5$.

6. For $\mathbb{Z}_2 \times \mathbb{Z}_5$, find the neutral additive element, the neutral multiplicative element, and all zero divisors.

7. Find the units of $\mathbb{Z}_2 \times \mathbb{Z}_5$; for each unit specify its inverse.

8. Determine the additive order of each element of $\mathbb{Z}_2 \times \mathbb{Z}_5$.

9. Compare the two rings $\mathbb{Z}_{10}$ and $\mathbb{Z}_2 \times \mathbb{Z}_5$ as best you can (we will learn tools to do this better, later in the course).

[1] This is an element x, such that $x \oplus y = y \oplus x = y$ for all y.

[2] This is an element x, such that $x \odot y = y \odot x = y$ for all y.

[3] This is a nonzero element x, such that there is some nonzero y with $x \odot y = 0$