1. Write the $⊕$-addition and $⊙$-multiplication tables of $\mathbb{Z}_{10}$.

2. For $\mathbb{Z}_{10}$, find the neutral additive element$^1$, the neutral multiplicative element$^2$, and all zero divisors$^3$.

3. Find the units of $\mathbb{Z}_{10}$; for each unit specify its inverse.

4. The *additive order* of an element in $\mathbb{Z}_{10}$ is the number of times one must $⊕$-add it to itself to get $[0]$. Determine the additive order of each element of $\mathbb{Z}_{10}$.

We define $\mathbb{Z}_2 \times \mathbb{Z}_5 = \{(a, b) : a \in \mathbb{Z}_2, b \in \mathbb{Z}_5\}$, the set of ordered pairs of elements, one each from $\mathbb{Z}_2$ and $\mathbb{Z}_5$. We define operations in the natural way, i.e. componentwise:

$$(a, b) ⊕ (a', b') = (a ⊕_2 a', b ⊕_5 b') \quad \text{and} \quad (a, b) ⊙ (a', b') = (a ⊙_2 a', b ⊙_5 b')$$

5. Write the $⊕$-addition and $⊙$-multiplication tables of $\mathbb{Z}_2 \times \mathbb{Z}_5$.

6. For $\mathbb{Z}_2 \times \mathbb{Z}_5$, find the neutral additive element, the neutral multiplicative element, and all zero divisors.

7. Find the units of $\mathbb{Z}_2 \times \mathbb{Z}_5$; for each unit specify its inverse.

8. Determine the additive order of each element of $\mathbb{Z}_2 \times \mathbb{Z}_5$.

9. Compare the two rings $\mathbb{Z}_{10}$ and $\mathbb{Z}_2 \times \mathbb{Z}_5$ as best you can (we will learn tools to do this better, later in the course).

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$^1$This is an element $x$, such that $x ⊕ y = y ⊕ x = y$ for all $y$.

$^2$This is an element $x$, such that $x ⊙ y = y ⊙ x = y$ for all $y$.

$^3$This is a nonzero element $x$, such that there is some nonzero $y$ with $x ⊙ y = 0$.