1. Prove that $(6, 15, 27) = (3)$ in $\mathbb{Z}$.

2. Find all ideals of $\mathbb{Z}_{12}$. Determine which of these are principal, maximal, and prime.

3. Suppose $I, J$ are ideals of some ring $R$. Prove that $I \cap J$ and $I + J$ are both ideals of $R$.

4. Let $R$ be a field. Prove that its only ideals are $(0)$ and $R$.

5. Let $R$ be a ring, and $a \in R$. Set $I = \{b \in R : ab = 0\}$. Prove that $I$ is an ideal of $R$.

6. Calculate simple forms for the elements of the ideal $I = (6x, 10)$ in $R = \mathbb{Z}[x]$. Is it principal? Maximal? Prime?

7. Calculate simple forms for the elements of the ideal $I = (6x, 10x)$ in $R = \mathbb{Z}[x]$. Is it principal? Maximal? Prime?

8. Prove that $\mathbb{Z}/20\mathbb{Z} \cong \mathbb{Z}_{20}$. Some people prefer to write $\mathbb{Z}/20\mathbb{Z}$ instead of $\mathbb{Z}_{20}$.

9. Let $I, K$ be ideals in $R$, with $K \subseteq I$. Prove that $I/K = \{x + K : x \in I\}$ is an ideal in $R/K = \{x + K : x \in R\}$.