Math 254 Spring 2014 Exam 8

Please read the following directions:
Please print your name in the space provided, using large letters, as “First LAST”. Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Friday 4/18; for details see the syllabus. You will find this exam on the instructor’s webpage later today.

1. Carefully state the definition of the “standard vector space” $\mathbb{R}^n$. Give a set of two vectors from $\mathbb{R}^4$.

2. Let $F, G$ be the linear mappings on $\mathbb{R}^2$ defined by $F(x, y) = (y, x+y)$ and $G(x, y) = (0, 2x)$. Find formulas defining the mappings: (a) $F + G$; (b) $2F - G$; (c) $FG$; (d) $GF$; (e) $F^2$.
The remaining three problems are all in vector space $V = \mathbb{R}^2$, with function $F : V \to V$ given by $F(x, y) = (x + 2y, 3x + 4y)$.

3. Prove that $F$ satisfies the definition of a linear mapping.

4. Find the rank and nullity of $F$.

5. Determine whether $F$ is an isomorphism, and if so find its inverse.