Name:

Math 254 Fall 2014 Exam 6

Please read the following directions:
Please print your name in the space provided, using large letters, as "First LAST". Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Wednesday 10/22; for details see the syllabus. You will find this exam on the instructor's webpage later today.

1. Carefully state the definition of "polynomial space" $P(t)$. Give two different bases for $P_1(t)$.

2. Let V denote the set of all symmetric 2×2 matrices. Set $E = \{\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)\}$. Prove that E is a basis for V.

The remaining three problems concern the vector space
$V = \{\left(\begin{smallmatrix} a & b \\ b & d \end{smallmatrix}\right) : a, b, d \in \mathbb{R}\}$ and its basis $E = \{\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)\}$.

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3. Set $B = \{\left(\begin{smallmatrix} 0 & -2 \\ -2 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)\} = \{b_1, b_2, b_3\}$. Compute $[b_1]_E, [b_2]_E, [b_3]_E$, and use these to prove that B is a basis for V.

4. Set $C = \{\left(\begin{smallmatrix} 1 & 3 \\ 3 & 2 \end{smallmatrix}\right), \left(\begin{smallmatrix} 2 & 6 \\ 6 & 4 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 1 \\ 1 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 5 & 3 \\ 3 & 4 \end{smallmatrix}\right)\} = \{c_1, c_2, c_3, c_4\}$. Compute $[c_1]_E, [c_2]_E, [c_3]_E, [c_4]_E$, and use these to find a basis for $Span(C)$.

5. For B as in (3), calculate Q_{BE}, and use this to compute $[\left(\begin{smallmatrix} 1 & 2 \\ 2 & 3 \end{smallmatrix}\right)]_B$.