Name:

Math 254 Fall 2013 Exam 6

Please read the following directions:
Please print your name in the space provided, using large letters, as “First LAST”. Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Friday 10/25; for details see the syllabus. You will find this exam on the instructor’s webpage later today.

1. Carefully state the definition of “standard vector space”. Give a vector from $\mathbb{R}^3$ and a dependent set of two vectors from $\mathbb{R}^4$.

2. Suppose that V is an n-dimensional vector space with basis S. Suppose that $\{v_1, v_2, \ldots, v_k\}$ is independent in V. Prove that $\{[v_1]_S, [v_2]_S, \ldots, [v_k]_S\}$ is independent in $\mathbb{R}^n$.

3. In the vector space $\mathbb{R}^2$, set $S = \{(1,3),(2,5)\}$, a basis. Find the change-of-basis matrix from the standard basis E to S, and use this matrix to find $[(1,1)]_S$.

The remaining two questions concern the vector space $P_3(t)$. Let $v_1 = -t^3 + t^2 + t + 1$, $v_2 = t^3 + 2t^2 + 2t + 1$, $v_3 = 4t^3 + 3t^2 + 3t + 1$, $v_4 = 4t^3 + 4t^2 + 4t + 2$.

4. Find a basis for $S = Span(\{v_1, v_2, v_3, v_4\})$.

5. Let $u = t^3 + t^2 + t + 1$. Determine whether or not $u \in S = Span(\{v_1, v_2, v_3, v_4\})$.