Math 254 Fall 2011 Exam 8

Please read the following directions:

Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Monday 11/14; for details see the syllabus. You will find this exam on the instructor's webpage later today.
The Chapter 9 exam will be delayed until Friday 11/18.

1. Carefully state the definition of "linear mapping". Give two examples, each from $\mathbb{R}^2$ to itself.

2. Consider the mapping $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by $f(x, y, z) = (2z - x, x + 2y + 3z)$. Determine whether this is linear.

3. Consider the linear map $f : \mathbb{R}^2 \to \mathbb{R}^3$ given by $f(x, y) = (x - 2y, 0, 4y - 2x)$. Find a basis for its kernel, and find a basis for its image.

4. Consider the linear map $\frac{d}{dt} : P_3(t) \to P_3(t)$. Find a basis for its kernel, and find a basis for its image.

5. Consider all possible linear mappings from $P_2(t)$ to $\mathbb{R}^1$. What are the possible nullities and ranks of these? Given an example for each possible combination, and indicate which are one-to-one and which are onto.