Math 254 Fall 2011 Exam 1 Solutions

1. Carefully state the definition of "subspace". Give two examples in $\mathbb{R}^2$.

   A subspace is a subset of a vector space, that is itself a vector space. Many examples are possible, including $\mathbb{R}^2$ itself, the zero-dimensional subspace $\{(0,0)\}$, and one-dimensional subspaces like $\{k(2,3) : k \in \mathbb{R}\}$.

2. Let $u = \begin{bmatrix} 5 & 6 & 12 \end{bmatrix}$, and $v = \begin{bmatrix} 4 \\ 0 \\ -2 \end{bmatrix}$. For each of the following, determine what *type* they are (undefined, scalar, matrix/vector). For each matrix/vector, specify the dimensions. **DO NOT CALCULATE ANY NUMBERS.**

   (a) $(u + v^T)^T$: 3×1 matrix or column vector
   (b) uvu: 1×3 matrix or row vector
   (c) $u^T vu$: undefined
   (d) $u \cdot (u \times v)$: scalar
   (e) $u \times (u \cdot v)$: undefined

3. Let $u = (1, 1, -1)$, $v = (4, 5, 10)$. Determine, with justification, whether these vectors (in $\mathbb{R}^3$) are orthogonal.

   We calculate $u \cdot v = 4 + 5 - 10 = -1$. Since $u \cdot v \neq 0$, these vectors are not orthogonal.

4. For $A = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$, calculate AB and BA.

   $AB = \begin{bmatrix} 1 & 1 \\ -10 & 1 \end{bmatrix}$, $BA = \begin{bmatrix} 0 & 5 & 10 \\ 2 & 1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$.

5. For $\bar{u} = (1, 1, -1)$ and $\bar{v} = (2, -1, 0)$, find $\bar{u} \times \bar{v}$ and $\bar{v} \times \bar{u}$.

   For $\bar{u} \times \bar{v}$, we consider $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} & \hat{\imath} & \hat{\jmath} \\ 1 & 1 & -1 & 1 & 1 \\ 2 & -1 & 0 & 2 & -1 \end{vmatrix}$, getting $(0\hat{\imath} - 2\hat{\jmath} - \hat{k}) - (2\hat{k} + \hat{\imath} + 0\hat{\jmath}) = -\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$, so $\bar{u} \times \bar{v} = (-1, -2, -3)$. For $\bar{v} \times \bar{u}$, we consider $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} & \hat{\imath} & \hat{\jmath} \\ 2 & -1 & 0 & 2 & -1 \\ 1 & 1 & -1 & 1 & 1 \end{vmatrix}$, getting $(\hat{\imath} + 0\hat{\jmath} + 2\hat{k}) - (-\hat{k} + 0\hat{\imath} - 2\hat{\jmath}) = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, so $\bar{v} \times \bar{u} = (1, 2, 3)$.

   ALTERNATE SOLUTION: $\bar{u} \times \bar{v} = (\hat{\imath} + \hat{\jmath} - \hat{k}) \times (2\hat{\imath} - \hat{\jmath}) = 2(\hat{\imath} \times \hat{\imath}) - (\hat{\imath} \times \hat{\jmath}) + 2(\hat{\jmath} \times \hat{\imath}) - (\hat{\jmath} \times \hat{\jmath}) - 2(\hat{k} \times \hat{\imath}) + (\hat{k} \times \hat{\jmath}) = 0 - \hat{k} - 2\hat{k} + 0 - 2\hat{\jmath} - \hat{\imath} = -\hat{\imath} - 2\hat{\jmath} - 3\hat{k} = (-1, -2, -3)$. Also, $\bar{v} \times \bar{u} = (2\hat{\imath} - \hat{\jmath}) \times (\hat{\imath} + \hat{\jmath} - \hat{k}) = 2(\hat{\imath} \times \hat{\imath}) - (\hat{\jmath} \times \hat{\imath}) + 2(\hat{\imath} \times \hat{\jmath}) - (\hat{\jmath} \times \hat{\jmath}) - 2(\hat{\imath} \times \hat{k}) + (\hat{\jmath} \times \hat{k}) = 0 + \hat{k} + 2\hat{k} + 0 + 2\hat{\jmath} + \hat{\imath} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} = (1, 2, 3)$.