MATH 254: Introduction to Linear Algebra
Chapter 0: Fundamental Definitions of Linear Algebra

This course contains ten fundamental definitions which will be tested repeatedly. Please memorize them, as well as how to apply them to specific examples. The first three are simpler than the others.

1. A **linear function/combination*** is a function, in one or more variables, that is entirely some mixture of addition and multiplication by constants, AND no constant is added by itself.

Examples: $f(x,y) = 3x + 2y$, $g(a,b,c,d) = 2a - b + 0c + 3d$, $f(x,y) = 0x + 0y$, $3z$, $7z + 8w$
Non-examples: $f(x,y) = 3x + 2y + 2$, $f(x) = x^2$, $f(x,y) = xy + y$, $f(x) = e^x$, $\sin x$
Note: the "no constant is added by itself" condition is equivalent to $f(0,0,\ldots,0) = 0$.

2. A **linear equation** is an equation setting a linear combination equal to any constant.
Examples: $3x + 2y = 2$, $2a - b + 0c + 3d = 0$, $3z = 12$, $x_1 + x_2 - x_3 = -4$, $0x + 0y = 3$, $0x + 0y = 0$
Non-examples: $x^2 = 2$, $xy + y = 7$, $x + \sin(y) = 4$, $e^x = 0$

3. A linear equation/function/combination is called **nondegenerate** if it contains at least one variable with a coefficient that is nonzero. It is **degenerate** if the coefficient of every variable is zero.

Nondegenerate: $3x + 2y = 2$, $2a - b + 0c + 3d = 0$, $f(z) = 3z$, $g(x_1, x_2) = 7x_1 + 0x_2$, $7x + 3y$
Degenerate: $0x + 0y = 2$, $0a + 0b + 0c + 0d = 0$, $f(x,y) = 0x + 0y$, $0x - 2y$, 0

4. A **vector space** is a collection of:
- objects, called vectors, and
- constants, called scalars. (for us, almost always the real numbers $\mathbb{R}$)

These vectors and scalars must satisfy a variety of properties (to be studied later). The most important property is **closure**: every linear combination of vectors, must again be a vector in the vector space.

One may USE closure as any linear combination (e.g. $5u + 7v - 8w$ is again a vector). To PROVE closure is easier: one needs to prove only two specific types of linear combination, not all of them:
- For every vector v and every scalar a, av is a vector, "scalar multiplication"
- For every two vectors u, v their sum $u + v$ is a vector. "vector addition"

Important Example 1: $\mathbb{R}^2$ is ordered pairs of real numbers (the vectors). For $u = (1,2), v = (-1,-1)$, we have $2u + 3v = 2(1,2) + 3(-1,-1) = (2,4) + (-3,-3) = (-1,1)$, which is again a vector.
Important Example 2: $\mathbb{R}^3$ is ordered triples of real numbers, such as $(1,2,3)$.
Important Example 3: $\mathbb{R}^n$ is ordered lists of n real numbers.
Non-Example: Ordered pairs (a,b) where $a > b$ are NOT a vector space; although closed under vector addition, they are not closed under scalar multiplication: $-2(5,4) = (-10,-8)$, which does not satisfy $-10 > -8$.

5. A **linear mapping/transformation** is a function, in one variable, from (the vectors of) a vector space to another (possibly the same) vector space. This function f must satisfy two properties:
- For every vector v and every scalar a, $f(av) = af(v)$, and
- For every two vectors u, v, $f(u + v) = f(u) + f(v)$.

Examples: $f(x) = 2x$, rotation, stretching, matrix multiplication, differentiation
Non-examples: $f(x) = \sin(x)$, because $\sin(\pi/2 + \pi/2) = \sin(\pi) = 0 \neq 2 = \sin(\pi/2) + \sin(\pi/2)$.
$f(x) = e^x$, because $e^{0+0} = e^0 = 1 \neq 2 = e^0 + e^0$. $f(x) = x + 3$, because $(7+7)+3 = 17 \neq 20 = (7+3)+(7+3)$.

*The difference between "function" and "combination" is that a combination is a function with no specified name.

6. A **subspace** of a vector space is itself a vector space, contained within a bigger one. The vector space properties are all inherited (for free) from the larger vector space, except for closure.

Important Example 1: The set of all linear combinations of any set of vectors. (the "span" of this set)
Important Example 2: The range of any linear mapping. (the "image" of this mapping)
Important Example 3: The set of vectors that a linear mapping sends to 0. (the "kernel" of this mapping)
Example 4: Consider the set S of all $v = (v_1, v_2)$, where $v_1 + v_2 = 0$. This is a subset of $\mathbb{R}^2$. For any scalar a and any vector v in S, $av = a(v_1, v_2) = (av_1, av_2)$, and $av_1 + av_2 = a(v_1 + v_2) = a(0) = 0$, so av is in S (first closure property). For any u, v in S, $u + v = (u_1, u_2) + (v_1, v_2) = (u_1 + v_1, u_2 + v_2)$, and $u_1 + v_1 + u_2 + v_2 = (u_1 + u_2) + (v_1 + v_2) = 0 + 0 = 0$, so $u + v$ is in S (second closure property). Hence S is a subspace.
Example 5: Consider the set T of all $v = (v_1, 0)$, another subset of $\mathbb{R}^2$. For any scalar a and any vector v in T, $av = a(v_1, 0) = (av_1, 0)$, which is in T (first closure property). For any u, v in T, $u+v = (u_1, 0)+(v_1, 0) = (u_1 + v_1, 0)$, which is in T (second closure property). Hence T is a subspace.
Non-example: Consider the set U of all $v = (v_1, 7)$, yet another subset of $\mathbb{R}^2$. This is not closed because $2v = (2v_1, 14)$ is not in U. The first closure property fails for $a = 2$ (and so is not true for all a and all v).

7. A set of vectors is called **dependent** if there is a nondegenerate linear combination of those vectors that yields the zero vector. Otherwise, the set of vectors is called **independent** – if EVERY nondegenerate linear combination of that set yields a NONzero vector.

Dependent: $x = (1, 1), y = (1, 2), z = (1, 3)$. The zero vector can be expressed with the nondegenerate function $f(x, y, z) = x - 2y + z = (1, 1) - 2(1, 2) + (1, 3) = (0, 0)$.
Independent: $x = (1, 1), y = (0, 1)$. The function $f(x, y) = 0x + 0y = (0, 0)$, but this is degenerate. Consider any linear function $f(x, y) = ax + by = a(1, 1) + b(0, 1) = (a, a + b)$. If this equals $(0, 0)$, then $a = b = 0$. Hence the zero vector cannot be expressed by a nondegenerate linear function on x, y.

8. A set of vectors is called **spanning** if every vector in the vector space can be expressed as a linear combination of the elements of that set.

Example 1: $\{(1, 0), (0, 1)\}$ spans $\mathbb{R}^2$. For every v in $\mathbb{R}^2$, $v = (v_1, v_2) = v_1(1, 0) + v_2(0, 1)$.
Example 2: $\{(1, 0), (0, 1), (0, 4)\}$ spans $\mathbb{R}^2$. For every v in $\mathbb{R}^2$, $v = (v_1, v_2) = v_1(1, 0) + v_2(0, 1) + 0(0, 4)$.
Example 3: $\{(1, 1), (0, 1)\}$ spans $\mathbb{R}^2$. For every v in $\mathbb{R}^2$, $v = (v_1, v_2) = v_1(1, 1) + (v_2 - v_1)(0, 1)$.
Non-example: $\{(1, 1)\}$ does not span $\mathbb{R}^2$. The linear combinations of this set all have their two coordinates equal, and so $(2, 3)$, to pick just one example, cannot be expressed in this way.

9. A **basis** is a set of vectors that is both spanning and independent. Equivalently, it is a maximal set of independent vectors ("maximally independent"). Equivalently, it is a minimal set of spanning vectors ("minimally spanning").

Important Example 1: The standard basis for $\mathbb{R}^n$ is $\{e_1, e_2, \ldots, e_n\}$, where e_i has a 1 in the i^{th} position and 0 in every other position. e.g. $\mathbb{R}^2$ has basis $\{e_1, e_2\} = \{(1, 0), (0, 1)\}$.
Example 2: $\{(1, 1), (0, 1)\}$ was shown above to be both independent and spanning. Thus it is a basis for $\mathbb{R}^2$.

10. The **dimension** of a vector space is the number of elements of a basis. It turns out that this number is the same, no matter what basis you choose.

Important Example 1: $\mathbb{R}^n$ has dimension n.
Important Example 2: The set of all linear combinations of k linearly independent vectors has dimension k.
Example 3: Consider the set of all $v = (v_1, v_2)$ where $v_1 + v_2 = 0$. This is a subspace, S, of $\mathbb{R}^2$. Its dimension cannot be 2 (since it is not all of $\mathbb{R}^2$), nor 0 (since it contains nonzero vectors), hence must be 1. Consider the set $\{(3, -3)\}$, drawn from S. It is independent (any single nonzero vector is independent by itself). It must be maximal, since otherwise the dimension of S would not be 1. Thus $\{(3, -3)\}$ is a basis for S.

Solved Problems

1. Carefully state the definition of "Linear Function".

   A linear function is a function that combines addition and multiplication by constants ONLY, and no constant is added by itself.

2. Carefully state the definition of "Degenerate Function".

   A linear function is degenerate if it is always zero no matter what it is evaluated on.

3. Carefully state the definition of "Linear Transformation".

   A linear transformation is a one-variable vector function f that satisfies $f(av) = af(v)$ and $f(u+v) = f(u) + f(v)$ (for every scalar a, and all vectors u, v).

4. Carefully state the definition of "Subspace".

   A subspace is a vector space that is contained within a larger vector space.

5. Carefully state the definition of "Basis".

   Solution 1: A basis is a set of vectors that is both spanning and independent.
   Solution 2: A basis is a maximal set of independent vectors.
   Solution 3: A basis is a minimal set of spanning vectors.

6. Determine which of the following equations is linear (justify your answers). Which are degenerate?
   A: $0x + 3y = 2y - 7$, B: $0x + 0y + 0z = 7$, C: $3x + 0xy = 7y$, D: $x/y = 3$

   A: This equation is equivalent to $0x + 1y = -7$ (or $1y = -7$), a linear combination set equal to a constant. Hence, it is linear. It is nondegenerate since the coefficient of y is 1.
   B: This equation is a linear combination set equal to a constant. Hence, it is linear, but degenerate. There are no (x, y, z) that satisfy this equation, incidentally.
   C: The equation is equivalent to $3x - 7y = 0$, hence it is linear, and nondegenerate. (the $0xy$ term does not ruin the linearity because of the 0).
   D: The x/y term DOES ruin the linearity. Although it is possible to multiply both sides by y to get $x = 3y$, a linear equation, the two equations are not equivalent. $x = y = 0$ satisfies $x = 3y$, but does not satisfy $x/y = 3$, so the two equations are (subtly) different. Since it is not linear, it is not meaningful to ask whether it is degenerate.

7. Consider the vector space $\mathbb{R}^3$, and set $v = (-3, 2, 0), u = (0, 1, 4)$. Calculate $2v - u$.

   $2v - u = 2(-3, 2, 0) - (0, 1, 4) = (-6, 4, 0) + (0, -1, -4) = (-6, 3, -4)$

8. Consider the vector space $\mathbb{R}^2$ and the vector function $f(x) = f((x_1, x_2)) = (2x_2, x_1)$. Determine whether or not f is a linear transformation.

   We calculate $f(ax) = f((ax_1, ax_2)) = (2ax_2, ax_1) = a(2x_2, x_1) = af(x)$. Since a, x were arbitrary, $f(ax) = af(x)$ holds for every scalar a and every vector x. We now calculate $f(x+y) = f((x_1+y_1, x_2+y_2)) = (2(x_2+y_2), (x_1+y_1)) = (2x_2+2y_2, x_1+y_1) = (2x_2, x_1) + (2y_2, y_1) = f(x)+f(y)$. Since x, y were arbitrary, $f(x+y) = f(x)+f(y)$ holds for all vectors x, y. Hence f is a linear transformation.

9. Consider the vector space $\mathbb{R}^2$ and the vector function $f(x) = f((x_1, x_2)) = (x_1x_2, 0)$. Determine whether or not f is a linear transformation.

   We calculate $f(ax) = f((ax_1, ax_2)) = (ax_1ax_2, 0) = a^2(x_1x_2, 0)$. This doesn't look like $af(x)$, so let's find a specific a, x as counterexample. For example, try $a = 2, x = (1, 1)$. $f(ax) = f((2, 2)) = (4, 0)$, whereas $af(x) = 2f((1, 1)) = 2(1, 0) = (2, 0)$. Hence f is NOT a linear transformation.

10. Consider the set S of all $v = (v_1, v_2)$ such that $|v_1| \geq |v_2|$. This is a subset of $\mathbb{R}^2$. Is it a subspace?

    For any scalar a and any vector v in S, we calculate $av = a(v_1, v_2) = (av_1, av_2)$. Because $|v_1| \geq |v_2|$, we may multiply both sides by the nonnegative $|a|$ to get $|a||v_1| \geq |a||v_2|$ and

hence $|av_1| \geq |av_2|$. Hence av is a vector in S; the first closure property holds.
We now take two vectors u, v in S, and calculate $u+v = (u_1, u_2)+(v_1, v_2) = (u_1+v_1, u_2+v_2)$. Must $|u_1 + v_1| \geq |u_2 + v_2|$? Not necessarily, so we need a specific counterexample. Many are possible, for example $u = (3, 1), v = (-3, 1)$. Both of u, v are in S, but $u + v = (0, 2)$ is not. Hence the second closure property does NOT hold. S is not a subspace, since to be a subspace both closure properties must hold.

11. Consider the vector space $\mathbb{R}^2$ and the linear transformation $f(x) = f((x_1, x_2)) = (2x_2, x_1)$. Consider the set of vectors that f sends to 0 (the "kernel" of f). Find this set, and verify that it is a subspace.

    If $f(x) = 0$, then $(2x_2, x_1) = (0, 0)$ and hence $2x_2 = 0, x_1 = 0$. Hence the kernel is just the single vector $(0, 0)$. This is indeed a subspace, since $a(0, 0) = (0, 0)$ and $(0, 0)+(0, 0) = (0, 0)$ (hence it is closed).

12. Consider the vector space $\mathbb{R}^2$ and the linear transformation $f(x) = f((x_1, x_2)) = (x_1+x_2, 0)$. Consider the set of vectors that f sends to 0 (the "kernel" of f). Find this set, and verify that it is a subspace.

    If $f(x) = 0$, then $(x_1 + x_2, 0) = (0, 0)$, and hence $x_1 + x_2 = 0$. Hence the kernel is all vectors (x_1, x_2) such that $x_1+x_2 = 0$. To verify it is a subspace, we must verify closure. We calculate $ax = a(x_1, x_2) = (ax_1, ax_2)$. Since $x_1 + x_2 = 0$, we must also have $ax_1 + ax_2 = 0$. Hence if x is in the kernel, ax is as well for every scalar a (first closure property). We calculate $x + y = (x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$. Since $x_1 + x_2 = 0$ and $y_1 + y_2 = 0$, in fact $x_1+y_1+x_2+y_2 = 0$. Hence if x, y are in the kernel, $x+y$ is a well (second closure property). Since both closure properties hold, this is indeed a subspace.

13. Consider the vector space $\mathbb{R}^2$, and set $u = (1, 1), v = (2, 3), w = (0, 5)$. Determine whether or not $\{u, v, w\}$ is dependent (justify your answer).

    Solution 1: $10u - 5v + w = 10(1, 1) - 5(2, 3) + (0, 5) = (10, 10) - (10, 15) + (0, 5) = (0, 0)$, so $f(u, v, w) = 10u - 5v + w$ is a nondegenerate function on $\{u, v, w\}$ that yields zerot. Hence, $\{u, v, w\}$ is dependent.
    Solution 2: If $\{u, v, w\}$ were independent, any basis of $\mathbb{R}^2$ would have at least three vectors. However, $\mathbb{R}^2$ has dimension 2, so this is impossible. Therefore, $\{u, v, w\}$ must be dependent.

14. Consider the vector space $\mathbb{R}^2$, and set $u = (2, 2), v = (3, 0)$. Determine whether or not $\{u, v\}$ is dependent (justify your answer).

    Solution 1: Suppose that $\{u, v\}$ were dependent. Then, there is some nondegenerate linear function yielding the zero vector. That is, there are some constants a, b (not both zero) so that $au + bv = (0, 0)$. We calculate $au + bv = a(2, 2) + b(3, 0) = (2a, 2a) + (3b, 0) = (2a + 3b, 2a) = (0, 0)$. So, we must have $2a + 3b = 0$ and $2a = 0$. The second equation gives us $a = 0$; we plug that into the first equation and get $b = 0$. Hence, $a = b = 0$ and the linear function was actually degenerate. Therefore, there is no nondegenerate linear function giving the zero vector, and therefore $\{u, v\}$ is independent.
    Solution 2: If $\{u, v\}$ were dependent, the set of all linear combinations would be a subspace of dimension 1. In this case, u would be a scalar multiple of v; but it is not since all scalar multiples of v have a 0 in the second coordinate. WARNING: this type of solution ONLY works for subspaces of dimension 1 – compare with the next problem.

15. Consider the vector space $\mathbb{R}^3$, and set $u = (1, 1, 1), v = (-1, 0, 1), w = (1, 2, 3)$. Determine whether or not $\{u, v, w\}$ is dependent (justify your answer).

    Suppose that $\{u, v, w\}$ were dependent. Then, there is some nondegenerate linear function giving the zero vector as output. That is, there are some constants a, b, c (not all zero) so that $au+bv+cw = (0, 0, 0)$. We calculate $au+bv+cw = (a, a, a)+(-b, 0, b)+(c, 2c, 3c) = (a-b+c, a+2c, a+b+3c) = (0, 0, 0)$. Hence $a-b+c = 0, a+2c = 0, a+b+3c = 0$. Adding the first and third equation gives $2a + 4c = 0$, which is equivalent to the second equation. There are thus infinitely many solutions, for example $a = 2, c = -1, b = 1$. We can double-check that

$2u+v-w = 2(1,1,1)+1(-1,0,1)-1(1,2,3) = (2,2,2)+(-1,0,1)+(-1,-2,-3) = (0,0,0)$. Hence this set is dependent.

NOTE: No one of u, v, w is a multiple of any one of the others. Although this was useful in the one-dimensional case (see the previous problem), the linear combinations of this set form a two-dimensional subspace and this approach is not helpful.

16. Consider the vector space $\mathbb{R}^2$, and set $u = (2,3)$. Determine whether or not $\{u\}$ is spanning (justify your answer).

    Solution 1: For it to be spanning, every $x = (x_1, x_2)$ could be expressed as a linear combination of u. Hence $(x_1, x_2) = a(2,3) = (2a, 3a)$. Hence $a = x_1/2$ and also $a = x_2/3$. Although this might be possible for some x, it need not hold always. For example, $x_1 = x_2 = 1$ has no possible a, so $(1,1)$ is not in the set of linear combinations of $\{u\}$. Hence this set is not spanning.

    Solution 2: Since $\mathbb{R}^2$ has dimension 2, any spanning set must have at least two elements. Hence this set is not spanning.

17. Consider the vector space $\mathbb{R}^2$, and set $u = (2,2), v = (3,0)$. Determine whether or not $\{u, v\}$ is spanning (justify your answer).

    Solution 1: For it to be spanning, every $x = (x_1, x_2)$ could be expressed as a linear combination of u, v. That is, there are scalars a, b with $x = au + bv = a(2,2) + b(3,0) = (2a+3b, 2a)$. If we set $a = x_2/2$ and set $b = (x_1-x_2)/3$ (both real numbers no matter what x is), the above holds. Hence this set is spanning.

    Solution 2: By an earlier problem this set is independent. But the dimension of $\mathbb{R}^2 = 2$, hence in fact this set is a basis. But then it is also spanning.

18. Consider the vector space $\mathbb{R}^2$, and set $u = (2,2), v = (3,0), w = (7,5)$. Determine whether or not $\{u, v, w\}$ is spanning (justify your answer).

    Comparing with the previous problem, every $x = (x_1, x_2) = au + bv$, for $a = x_2/2$ and $b = (x_1-x_2)/3$. Hence $x = au + bv + 0w$, a linear combination of $\{u, v, w\}$, so this set is also spanning.

19. Consider the vector space $\mathbb{R}^3$, and set $u = (1,1,1), v = (-1,0,1), w = (1,2,3)$. Determine whether or not $\{u, v, w\}$ is spanning (justify your answer).

    Solution 1: For this to be spanning, every $x = (x_1, x_2, x_3)$ would be some linear combination of u, v, w. Hence there needs to be a, b, c with $(x_1, x_2, x_3) = a(1,1,1)+b(-1,0,1)+c(1,2,3) = (a-b+c, a+2c, a+b+3c)$. This yields equations $a-b+c = x_1, a+2c = x_2, a+b+3c = x_3$. Can these always be solved, for every x_1, x_2, x_3? This is actually rather tricky to answer (we will learn more tools soon), but it turns out the answer is no. We need a specific counterexample – many are possible, for example $x_1 = x_2 = 1, x_3 = 0$. In this case, the three equations are $a-b+c = 1, a+2c = 1, a+b+3c = 0$. Adding the first and third equations gives $2a+4c = 1$, which is inconsistent with the second equation. Hence $x = (1,1,0)$ is not expressible as a linear combination of $\{u, v, w\}$, and this set is not spanning.

    Solution 2: The set $\{u, v, w\}$ has three elements, and $\mathbb{R}^3$ has dimension 3. Hence if this set were spanning, it would also be a basis and therefore independent. But an earlier problem showed that this set is not independent, hence it cannot be spanning.

20. Find three different bases for $\mathbb{R}^2$.

    Many solutions are possible. An easy choice is the standard basis $\{e_1, e_2\} = \{(1,0), (0,1)\}$. An earlier problem showed that $\{(2,2), (3,0)\}$ is another basis. An example given after the definition of spanning showed that $\{(1,1), (0,1)\}$ is spanning, and thus is a basis since it contains two elements.

Supplementary Problems

Be sure to thoroughly justify all your solutions.

21. Carefully state the definition of "Linear Equation".
22. Carefully state the definition of "Linear Combination".
23. Carefully state the definition of "Spanning".
24. Carefully state the definition of "Independent".
25. Carefully state the definition of "Basis".
26. For the vectors $u = (1, 2, 3), v = (4, 0, 1), w = (-3, -2, 5)$, calculate $2u - 3v - 4w$.
27. Determine which of the following functions is linear (justify your answers). A: $f(x, y) = 7x-3y+2x+4y$, B: $f(x, y) = 0x + 0y + 0$, C: $f(x, y) = 2x + 3y + 4$, D: $f(x, y, z) = (x/y)(y/z)z$, E: $f(x, y, z) = x$
28. Consider the vector space $\mathbb{R}^2$ and the vector function $f((x_1, x_2)) = (x_2, 0)$. Is this a linear mapping?
29. Consider the vector space $\mathbb{R}^2$ and the vector function $f((x_1, x_2)) = (x_1^3, 0)$. Is this a linear mapping?
30. Consider the set S of all vectors $v = (v_1, v_2)$ such that $2v_1 + v_2 = 0$. Determine whether or not this is a subspace of $\mathbb{R}^2$.
31. Consider the set S of all vectors $v = (v_1, v_2)$ such that $v_1 v_2 = 0$. Determine whether or not this is a subspace of $\mathbb{R}^2$.
32. Consider the vector space $\mathbb{R}^2$ and the linear mapping $f((x_1, x_2)) = (2x_1 + x_2, 0)$. Consider the set of vectors that f sends to 0 (the "kernel" of f). Find this set, and verify that it is a subspace.
33. Consider the vector space $\mathbb{R}^2$, and set $u = (2, 6), v = (-3, -9)$. Determine whether or not $\{u, v\}$ is independent.
34. Consider the vector space $\mathbb{R}^2$, and set $u = (2, 6), v = (-3, -9), w = (5, 15)$. Determine whether or not $\{u, v, w\}$ is independent.
35. Consider the vector space $\mathbb{R}^2$, and set $u = (2, 6), v = (0, -9)$. Determine whether or not $\{u, v\}$ is independent.
36. Consider the vector space $\mathbb{R}^2$, and set $u = (2, 6), v = (-3, -9)$. Determine whether or not $\{u, v\}$ is spanning.
37. Consider the vector space $\mathbb{R}^2$, and set $u = (2, 6), v = (-3, -9), w = (5, 15)$. Determine whether or not $\{u, v, w\}$ is spanning.
38. Consider the vector space $\mathbb{R}^2$, and set $u = (2, 6), v = (0, -9)$. Determine whether or not $\{u, v\}$ is spanning.
39. Which of the sets given in problems 36-38 are bases of $\mathbb{R}^2$?
40. For the sets given in problems 36-38, determine the dimension of the subspace they span.

Answers to Supplementary Problems: (WARNING: these are just answers, NOT thoroughly justified solutions)

26: $(2, 12, -17)$ 27: A,B,E 28: yes 29: no 30: yes 31: no 33: no 34: no 35: yes 36: no 37: no 38: yes 39: just 38 40: 1,1,2