Math 254-2 Exam 9: 11/25/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on the attached page only (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Tuesday 12/2; for details see the syllabus. Each problem is worth 10 points; your total will be doubled to fit the standard 100 point scale. You have approximately 30 minutes.

1. Carefully define the term “spanning”. Give two examples in $\mathbb{R}^2$.

2. Consider the basis $S = \{(1, 2), (2, 5)\}$ of $\mathbb{R}^2$, and the linear operator $F(x, y) = (2x - 3y, x - y)$. Find the matrix representation $[F]_S$.

3. Prove that, for any square matrices $A, B$, if $A$ is similar to $B$, then $B$ must be similar to $A$.

For the last two questions, set $V$ to be the vector space of functions that have as a basis $S = \{1, \sin \theta, \cos \theta, \sin 5\theta, \cos 5\theta\}$.

4. Let $D$ be the differential operator on $V$, $D(f(\theta)) = f'(\theta)$. Find the matrix representation $[D]_S$.

5. Let $L$ be the operator on $V$ given by $L(f(\theta)) = f''(\theta) - 2f(\theta)$. Find the matrix representation $[L]_S$.

Please hand in ONLY the second page; keep this first page.
ID Code: ________________

Please write all solutions on this page (front and back if necessary).