Math 254-1 Exam 8: 11/10/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on the attached page only (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Wednesday 11/12; for details see the syllabus. Each problem is worth 10 points; your total will be doubled to fit the standard 100 point scale. You have approximately 30 minutes.

1. Carefully define the linear algebra term “linear mapping”. Give two examples on \( \mathbb{R}^2 \).

2. Give any inner product on \( \mathbb{R}^2 \), OTHER than the dot product. Use your inner product to calculate \( \langle u, v \rangle \) for \( u = (1, 1)^T \), \( v = (2, 3)^T \).

3. Find two different functions \( f, g \) on \( \mathbb{R} \), with \( f \circ f = g \circ g = 1_\mathbb{R} \).

4. Consider all possible linear mappings from \( \mathbb{R}^4 \) to \( \mathbb{R}^2 \). What are the possible nullities and ranks of these? Give an example function for each possible combination, and indicate which functions are one-to-one and which are onto.

5. Consider the mapping \( F : \mathbb{R}_2[t] \to \mathbb{R}^2 \) given by \( F(p(t)) = (p(2), p(-1)) \). Calculate \( F(p(t)) \) for \( p(t) = t^2 + 3t - 1 \). Determine whether \( F \) is linear.

Please hand in ONLY the second page; keep this first page.
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Please write all solutions on this page (front and back if necessary).