Math 254-2 Exam 7: 11/4/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on the attached page only (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 11/6; for details see the syllabus. Each problem is worth 10 points; your total will be doubled to fit the standard 100 point scale. You have approximately 30 minutes.

1. Carefully define the linear algebra term “degenerate”. Give two examples.

2. Carefully define the linear algebra term “inner product”. Give two examples on \( \mathbb{R}^2 \).
   The remaining three problems all concern the vector space \( M_{2,2}(\mathbb{R}) \) with standard inner product \( <A, B> = \text{tr}(B^T A) \).

3. Find all values of \( k \) such that \( \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \) and \( \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \) are orthogonal.

4. Find a basis for \( \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \) \( \perp \).

5. Use the Gram-Schmidt process to find an orthogonal basis for the space \( \text{Span} \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} \right\} \).

Please hand in ONLY the second page; keep this first page.
ID Code: ________________

Please write all solutions on this page (front and back if necessary).