Math 254-1 Exam 4: 10/13/8
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on the attached page only (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Wednesday 10/15; for details see the syllabus. Each problem is worth 10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

1. Carefully state the definition of “subspace”. Give two examples from $\mathbb{R}^2$.

2. Carefully state five of the eight vector space axioms.

3. Let $S = \{f(x) : f(3) = 0\} \subseteq \mathbb{R}[x]$ be the set of all polynomials that are zero at $x = 3$. Prove that this is a vector space.

4. Determine, with justification, whether $(1,1,1)$ is in $\text{Span}(S)$, for $S = \{(1,2,3), (2,0,1), (-3, 2, 1)\}$.

5. Let $W_1 = \text{Span}(S)$, for $S = \{(1\ 1\ 0\ 0), (0\ 0\ 1\ 1)\}$. Let $W_2 = \text{Span}(T)$, for $T = \{(1\ 0\ 0\ 0), (0\ 1\ 0\ 0)\}$. Prove that $W_1 \oplus W_2 = M_{22}(\mathbb{R})$ (the set of all $2 \times 2$ matrices).

Please hand in ONLY the second page; keep this first page.
ID Code: ________________

Please write all solutions on this page (front and back if necessary).