Math 254 Exam 9: 11/28/6
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 11/30; for details see the syllabus. Each problem is worth 10 points.

1. Carefully define the term “basis”. Give two examples in \( \mathbb{R}^2 \).

2. Consider the basis \( S = \{(1, -2), (2, -5)\} \) of \( \mathbb{R}^2 \), and the linear operator \( F(x, y) = (2x + 3y, 4x - 5y) \). Find the matrix representation \([F]_S\).

3. Let \( V \) be the vector space of functions that have as a basis \( S = \{1, \sin \theta, \cos \theta, \sin 2\theta, \cos 2\theta\} \). Let \( D \) be the differential operator on \( V \). Find the matrix representation \([D]_S\).
   BONUS: What is the nullity and rank of \( D \)?

4. Set \( A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \). Find two other matrices similar to \( A \).

5. Prove that, for any square matrices \( A, B \), if \( A \) is similar to \( B \), then \( B \) must be similar to \( A \).