

### Math 254 Exam 9: 11/28/6

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 11/30; for details see the syllabus. Each problem is worth 10 points.

1. Carefully define the term “basis”. Give two examples in  $\mathbb{R}^2$ .
2. Consider the basis  $S = \{(1, -2), (2, -5)\}$  of  $\mathbb{R}^2$ , and the linear operator  $F(x, y) = (2x + 3y, 4x - 5y)$ . Find the matrix representation  $[F]_S$ .
3. Let  $V$  be the vector space of functions that have as a basis  $S = \{1, \sin \theta, \cos \theta, \sin 2\theta, \cos 2\theta\}$ . Let  $D$  be the differential operator on  $V$ . Find the matrix representation  $[D]_S$ .  
BONUS: What is the nullity and rank of  $D$ ?
4. Set  $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ . Find two other matrices similar to  $A$ .
5. Prove that, for any square matrices  $A, B$ , if  $A$  is similar to  $B$ , then  $B$  must be similar to  $A$ .