Math 254 Exam 8: 11/14/6

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 11/16; for details see the syllabus. Each problem is worth 10 points.

NOTE: This is a group exam; you are encouraged to collaborate with your classmates. You have approximately 45 minutes.

1. Carefully define the term "linear mapping". Give two examples in $\mathbb{R}^2$.

2. Consider the mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $f(x, y, z) = (x + 2y, 0, -3y)$. Is this linear?

3. Consider all relations whose domain is $\{A, B\}$ and whose codomain is $\{1, 2, 3\}$. For each of the following, either give an example or state that no example exists.

   (a) A one-to-one function.

   (b) An onto function.

   (c) A function that is neither one-to-one nor onto.

   (d) A relation whose inverse is a function.

   BONUS: How many relations are there on this domain/codomain combination? How many functions are there?

4. Consider the linear mapping $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x, y) = (x - y, x - 2y)$. Find a formula for f^{-1}.

5. Consider all linear mappings from $\mathbb{R}^3$ to $\mathbb{R}^2$. What are the possible nullities? What are the possible ranks? Give specific examples illustrating each possible value.