Math 254 Exam 8: 11/14/6
Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 11/16; for details see the syllabus. Each problem is worth 10 points.

NOTE: This is a group exam; you are encouraged to collaborate with your classmates. You have approximately 45 minutes.

1. Carefully define the term “linear mapping”. Give two examples in $\mathbb{R}^2$.

2. Consider the mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $f(x, y, z) = (x + 2y, 0, -3y)$. Is this linear?

3. Consider all relations whose domain is $\{A, B\}$ and whose codomain is $\{1, 2, 3\}$. For each of the following, either give an example or state that no example exists.
   (a) A one-to-one function.
   (b) An onto function.
   (c) A function that is neither one-to-one nor onto.
   (d) A relation whose inverse is a function.

   BONUS: How many relations are there on this domain/codomain combination? How many functions are there?

4. Consider the linear mapping $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $f(x, y) = (x - y, x - 2y)$. Find a formula for $f^{-1}$.

5. Consider all linear mappings from $\mathbb{R}^3$ to $\mathbb{R}^2$. What are the possible nullities? What are the possible ranks? Give specific examples illustrating each possible value.