## Math 254 Exam 2a Solutions

1. Carefully state the definition of "linear equation". Give two examples, one in standard form and one NOT in standard form.

   A linear equation has multiplication by constants and addition ONLY. Constants may appear alone. $3x + 2y = 7$ is in standard form. $3x + 4y - 7 = 0$ is not.

2. Solve the following system, using back-substitution. Show your work.

$$\begin{aligned} 3x_1 - 5x_2 + 2x_3 + 2x_4 &= -3 \quad (A) \\ 6x_2 - 8x_3 - 3x_4 &= -3 \quad (B) \\ 6x_3 - x_4 &= -5 \quad (C) \\ 3x_4 &= 15 \quad (D) \end{aligned}$$

   First, we solve (D) to get $x_4 = 5$. Second, (C) becomes $6x_3 - 5 = -5$, which we solve to get $x_3 = 0$. Third, (B) becomes $6x_2 - 8 \times 0 - 3 \times 5 = -3$, which we solve to get $x_2 = 2$. Finally, (A) becomes $3x_1 - 5 \times 2 + 2 \times 0 + 2 \times 5 = -3$, which we solve to get $x_1 = -1$. To summarize, $(x_1, x_2, x_3, x_4) = (-1, 2, 0, 5)$ is the only solution.

3. Give three examples of $2 \times 2$ systems of linear equations. One should have no solutions, one should have one solution, and one should have infinitely many solutions. Justify your answers, providing all solutions to each system.

   Many, many possible answers exist. $\{x = 1, x+y = 3\}$ has one solution: $\{x = 1, y = 2\}$. $\{x+y = 2, x+y = 1\}$ has no solutions, since we can subtract the second equation from the first to get $0 = 1$, which is never true. $\{x+y = 1, 2x+2y = 2\}$ has infinitely many solutions, since the second equation is twice the first. Its infinitely many solutions can be given as $\{x = a, y = 1 - a : a \text{ any real number}\}$.

4. Find the line of best fit for the following set of points: $\{(-1, 0), (0, 3), (2, 5)\}$.

   We find $N = 3, \Sigma x = 1, \Sigma y = 8, \Sigma x^2 = 5, \Sigma xy = 10$. Therefore, we need to solve $\{3b + m = 8, b + 5m = 10\}$. Add -3 times the second equation to the first equation to get $-14m = -22$. We solve to get $m = 22/14 = 11/7$. We plug into $b+5m = 10$ to get $b + 55/7 = 70/7$; hence $b = 15/7$. Therefore, the line of best fit is $y = 11/7x + 15/7$.

5. Solve the following system of linear equations.

$$\begin{aligned} 4x - 3z &= 0 \\ -2x + 3y + 2z &= -1 \\ 6x - 6y - 6z &= 1 \end{aligned}$$

$$\left[\begin{array}{rrr|r} 4 & 0 & -3 & 0 \\ -2 & 3 & 2 & -1 \\ 6 & -6 & -6 & 1 \end{array}\right] \to \left[\begin{array}{rrr|r} 4 & 0 & -3 & 0 \\ 0 & 3 & 1/2 & -1 \\ 0 & -6 & -3/2 & 1 \end{array}\right] \to \left[\begin{array}{rrr|r} 4 & 0 & -3 & 0 \\ 0 & 3 & 1/2 & -1 \\ 0 & 0 & -1/2 & -1 \end{array}\right]$$

   $R2 + (1/2)R1 \to R2, \quad R3 + (-3/2)R1 \to R3, \quad R3 + (2)R2 \to R3$

   $(-1/2)z = -1$; hence $z = 2$. $3y + (1/2)2 = -1$; hence $y = -2/3$. $4x - (3)2 = 0$; hence $x = 3/2$. To summarize $(x, y, z) = (3/2, -2/3, 2)$ is the only solution.