1. (Question 1 is just instructions; this is a weird requirement of Gradescope)
2. Prove that \( \forall x \in \mathbb{R}, \exists! y \in \mathbb{R}, (x = \lfloor x \rfloor + y) \land (0 \leq y < 1). \)
3. Use the division algorithm to prove that \( \forall n \in \mathbb{N}, \frac{n^2 + 9n + 20}{2} \in \mathbb{Z}. \)
4. Use (some form of) mathematical induction to prove that \( \forall n \in \mathbb{N}, \frac{n^2 + 9n + 20}{2} \in \mathbb{Z}. \)
5. Solve the recurrence given by \( a_0 = 2, \ a_1 = 3, \ a_n = -4a_{n-1} - 4a_{n-2} \ (n \geq 2). \)
6. Let \( a_n = n^{1.9} + n^{2.1}. \) Prove or disprove that \( a_n = O(n^2). \)
7. Let \( F_n \) denote the Fibonacci numbers. Prove that \( \forall n \in \mathbb{N}_0, \ F_{2n+1}^2 - F_{2n+2}F_{2n} = 1. \)