1. Consider \( f : \mathbb{R} \rightarrow \mathbb{R} \) given by \( f(x) = x\lfloor x \rfloor \). Prove or disprove that \( f \) is injective.

   False. We have \( f(0) = 0 \lfloor 0 \rfloor = 0 = \frac{1}{2} \lfloor \frac{1}{2} \rfloor = f\left(\frac{1}{2}\right) \), but \( 0 \neq \frac{1}{2} \).

2. Let \( A, B, C \) be sets, with \( B \subseteq C \). Prove that \( (A \times B) \subseteq (A \times C) \).

   Let \( x \in A \times B \) be arbitrary. There must be some \( a \in A, b \in B \) such that \( x = (a, b) \).
   Since \( B \subseteq C \), in fact \( b \in C \). Hence \( x = (a, b) \in A \times C \). Therefore \( (A \times B) \subseteq (A \times C) \).

3. Carefully define each of the following terms:
   a. relation

      A relation from set \( A \) to set \( B \) is a subset of \( A \times B \).

   b. symmetric (relation)

      A relation \( R \) is symmetric if whenever \( (a, b) \in R \), we must have \( (b, a) \in R \).

   c. equivalence relation

      A relation is an equivalence relation if it is reflexive, symmetric, and transitive.

   d. partial order

      A relation is a partial order if it is reflexive, antisymmetric, and transitive.

   e. surjective

      A function \( f : A \rightarrow B \) is surjective if for every \( b \in B \) there is at least one \( a \in A \) such that \( f(a) = b \).

4. Consider the relation \( R \) on \( \mathbb{Z} \) given by \( aRb \iff |a - b| \leq 1 \). Prove or disprove that \( R \) is transitive.

   False. We have \( 3R2 \) since \( |3 - 2| \leq 1 \). We have \( 2R1 \) since \( |2 - 1| \leq 1 \). But \( 3R1 \) since \( |3 - 1| > 1 \).

5. Find the general solution to the recurrence relation \( a_n = -a_{n-1} + 6a_{n-2} \).

   This relation has characteristic equation \( r^2 = -r + 6 \), which rearranges as \( r^2 + r - 6 = 0 \),
   and factors as \( (r + 3)(r - 2) = 0 \). There are two roots, \(-3\) and \(2\), so the general solution is \( a_n = A(-3)^n + B(2)^n \).